Forecaster (Mis-)Behavior*

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Abstract

We document two stylized facts in expectational survey data. First, professional forecasters' *overrevise* their macroeconomic expectations. Second, such overrevisions mask evidence of both *over- and underreactions* to salient public signals. We show that the first fact is inconsistent with standard models of noisy rational expectations, but consistent with behavioral and strategic models of forecaster behavior. The second fact, in contrast, presents a puzzle for existing theories of expectation formation. To explain this evidence, we propose a simple extension of noisy rational expectations that allows forecasters to be overconfident in their information. We show that this feature, when combined with the endogeneity of public signals, leads forecasters to over- and undereact to public information in a manner that is consistent with the data. Lastly, we validate our model by studying its implications for the precision and dispersion of forecasts, and discuss the conditions under which forecasters over- and underreact to new information.

JEL codes: C53, D83, D84, E31 *Keywords:* Expectations, forecasters, information

1 Introduction

Expectations are central to economics. Because individual expectations are typically unobserved, however, it is often difficult to discriminate between alternative models of expectation formation. One exception are forecaster surveys, which regularly publish individual expectations about macroeconomic and financial variables. Indeed, Muth (1961) proposed the rational

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expectations theory in part to explain the perceived sluggishness of survey expectations as a rational response to noisy information.[1]

Although the full information variant of rational expectations later became the benchmark of modern macroeconomics, the work of Woodford (2002), Sims (2003), and others,[2] has revived interest in noisy information models of rational expectations. In turn, this has rekindled interest in the use of survey data to better discipline and test such models. In line with a central prediction of noisy rational expectations, Coibion and Gorodnichenko (2012, 2015) recently document that the average of survey expectations underreacts to new information relative to what a full information framework would prescribe.[3] However, such underreactions of *average* expectations are not only consistent with noisy rational expectations, but also with a host of other both rational and behavioral theories of *individual* expectation formation.

In this paper, we provide new evidence on the statistical properties of individual survey expectations of macroeconomic variables. We document two stylized facts that present a challenge for noisy rational expectations. First, individual forecasters' *overrevise* their macroeconomic expectations. Second, such overrevisions mask both *over- and underreactions* to salient public signals. We show that the first fact is inconsistent with standard models of noisy rational expectations, but in line with e.g. models of strategic forecaster behavior. The second fact, in contrast, presents a puzzle for existing theories of expectation formation.

To explain this evidence, we propose a simple extension of noisy rational expectations that allows forecasters to be overconfident in the precision of their own information (both relative to the truth and relative to their perception of others). We show that such overconfidence makes forecasters overrevise their expectations and misperceive others' responses to information. Importantly, such misperception leads forecasters to misinterpret public signals that aggregate others' actions, and results in over- or underreactions that are consistent with the data.

A well-known consequence of rational (mean-squared optimal) expectations is that individual forecast errors should be unpredictable based on known information. The two stylized facts that motivate our theory result from tests of this prediction. Our first test relates individual forecast errors to individual revisions in fixed-date forecasts.[4] Basic introspection by rational forecasters requires these two to be uncorrelated, even in the presence of noisy information. Our second test instead exploits the survey data to relate individual forecast errors directly to elements of public information that are salient to forecasters. Once more,

[1] See, for example, p. 316 in Muth (1961).

[2] See, for instance, Angeletos and Pavan (2007), Nimark (2008), Lorenzoni (2009), Maćkowiak and Wiederholt (2009), Paciello and Wiederholt (2013), and Angeletos *et al.* (2016).

[3] Coibion and Gorodnichenko (2012), Andrade and Le Bihan (2013), Fuhrer (2018), Bordalo *et al.* (2019), and Kohlhas and Walther (2018) document related evidence.

[4] In coinciding and independent work, Bordalo *et al.* (2019) propose a similar test. We discuss the similarities and differences between the two approaches in the related literature section.

individual rationality requires individual forecast errors to be uncorrelated with these.

As a benchmark, we first consider inflation expectations from the US Survey of Professional Forecasters (SPF). We focus on inflation forecasts to make our results comparable to previous studies, which have focused disproportionately on inflation. We use the outcomes of our tests to document two empirical results.

First, individual forecast revisions are systematically too large. This manifests itself in a pronounced negative relationship between individual forecast errors, on the one hand, and individual forecast revisions, on the other hand. Second, this observed overrevision masks evidence of both over- and underreactions to salient public signals that are both predictive about future inflation, relevant, and observed in real-time (e.g. previous consensus forecasts or changes in the unemployment rate). We document that these patterns extend to forecasts of other macroeconomic variables than inflation, different forecast horizons, to different countries, as well as to other forecasters than those that call themselves professional.

Combined, our empirical results present a challenge for existing models of expectation formation. While simple models of noisy rational expectations are consistent with an underrevision of average expectations, they are *prima facie* inconsistent with the overrevision documented at the individual level. Alternative theories of forecaster behavior that incorporate (i) the specific strategic considerations faced by professional forecasters (e.g. Laster *et al.*, 1999; Ottaviani and Sørensen, 2006; Ehrbeck and Waldmann, 1996); (ii) common behavioral biases (e.g. Daniel *et al.*, 1998; Bordalo *et al.*, 2019); or (iii) trembling-hand noise can naturally explain such overrevisions. However, as we show, these theories either all predict optimal use of public information (conditional on private information), or that forecasters overreact to *all* new information, irrespective of its source. Both predictions are inconsistent with the simultaneous over- and underreactions to public information that we document.[5]

To account for our empirical results, we propose a simple extension of noisy rational expectations that allows forecasters to be overconfident in their own information. Specifically, we allow forecasters to both perceive their private information to be more precise than it actually is ("*absolute overconfidence*"; Alpert and Raiffa, 1982; Soll and Klayman, 2004; and others), and to be more precise than the information of others' ("*relative overconfidence*"; Alicke and Govorun, 2005; Larrick *et al.*, 2007). Both dimensions of overconfidence are commonly used in the psychology literature (Moore and Healy, 2008) and are consistent with reported forecast densities in the SPF that systematically underestimate forecast errors. We show that, taken together, absolute and relative overconfidence can explain our empirical results when combined with the central fact that most public signals reflect the outcome of others' choices, and

[5] Angeletos and Huo (2019) show that the overrevisions that we document are also inconsistent with two common alternatives to noisy rational expectations: "cognitive discounting" (Gabaix, 2017) and "level-k thinking" (Farhi and Werning, 2019).

hence their information). The combination of overconfidence with the endogeneity of public signals further distinguishes our theory from previous models of overconfidence (e.g. Daniel *et al.*, 1998; Thaler, 2000).

All else equal, absolute overconfidence makes forecasters overreact to private information, and hence makes individuals forecast revisions too large. In contrast, relative overconfidence makes forecasters underestimate the precision of others' information. As a result, forecasters expect others to react less to their own information than warranted. This is important. In any model of expectation formation that accounts for the endogeneity of public information, forecasters need to form a view about the behavioral rules followed by others, to determine the informativeness of endogenous signals. By underestimating others' responses, relative overconfidence causes forecasters to expect public signals to respond less to new information, and to therefore be less precise.

Such misperception has two offsetting effects: Underestimating the precision of public signals, all else equal, leads forecasters to dismiss them, and underreact to their realizations. However, underestimating the responsiveness of public signals, by contrast, leads forecasters to over-infer information from any given signal realization, and hence to overreact.

We demonstrate these results within the context of a standard noisy rational expectations model. In the model, a continuum of forecasters with mean-squared error preferences attempt to repeatedly estimate an unobserved fundamental, using both private and endogenous public information. However, unlike similar models (see Vives, 2010; Veldkamp, 2011), forecasters exhibit absolute and relative overconfidence. We show that this extension amounts to a simple one-step deviation from noisy rational expectations.[6]

Although our model is simple, we quantitatively validate it along three dimensions. First, we show that our model can match the estimated overrevision of inflation forecasts at the same time as the estimated overreaction to a particular public signal, previous period's consensus forecast. We focus on consensus forecasts because it reflects a public signal that simply aggregates other's information. This allows us to focus on overconfidence's role in creating a friction between forecasters' perception of a public signal and that which arises in equilibrium. As argued in Ottaviani and Sørensen (2006), consensus also represents a particularly salient public signal for professional forecasters, such as those in the SPF.[7]

Second, an attractive feature of the survey data on professional forecasters is that respondents also report forecast densities, in addition to point estimates. We show that this additional information allows us to directly test and validate our assumptions of absolute and relative overconfidence in the survey data.

[6] The model thus falls within the "portable extensions of known models" class (Rabin, 2013).

[7] Estimates of responses to consensus can further be used to directly discipline social learning models (e.g. Banerjee, 1992; Vives, 1997).

Lastly, two key implications of our model are that (i) forecasters should underreact relatively more to public signals that are less precise, and (ii) that the magnitude of over- and underreactions should change with the volatility of the forecasted variable. We demonstrate that both predictions are in line with the patterns of responses that we document in the data. In particular, our model accurately predicts the changes in the magnitudes of over- and underreactions before and after the Great Moderation.

We conclude the paper by studying the implications of our model for the distribution of forecast errors. Although overconfidence implies unnecessary losses relative to the rational benchmark, these are largely offset by more informative public signals. Equilibrium forecast errors are of a similar magnitude to those from the rational model, making it more difficult for individual forecasters to detect their overconfidence. This, in effect, is because overconfidence in private information internalizes the learning externality that exists in markets with endogenous public signals (e.g Amador and Weill, 2010), and that otherwise causes forecasters to attach too little weight to private information. Individually, the combination of absolute and relative overconfidence may therefore be close to "group-optimal" (Smith, 1982).

Finally, admittedly, professional forecasters may differ from other economic agents in their incentives and information about the state of the economy. In this paper, we confront this issue by directly contrasting the ability of agency-based models to explain the observed under- and overreactions with simple behavioral alternatives. To the extent that the evidence we uncover below speaks in favor of widely documented behavioral biases, rather than particular strategic incentives, we think that our results should carry over to other contexts. Indeed, we provide some evidence to this effect later in the paper.

Related Literature: Our paper is related to several strands of research. We review these in order of proximity, starting with the most closely related and ending with the substantial body of work that links over- and underreactions of expectations to asset price anomalies.

First, our paper relates to studies that use expectational survey data to test noisy rational expectations. Muth (1961), Zarnowitz and Lambros (1987), and Keane and Runkle (1990) discuss how the precision of average forecasts exceeds the precision of individual forecasts, as well as how the distribution of forecasts is substantially dispersed.[8] Both features are consistent with models in which forecasters observe noisy, private (instead of full) information.

More recently, Coibion and Gorodnichenko (2015), and others,[9] show that average forecasts of several macroeconomic variables, across different surveys, underreact to new information, in the sense that average forecast revisions are positively correlated with forecast errors. Our study departs from this observation, and studies both average and individual-level forecasts

[8] Pesaran (1987) and Pesaran and Weale (2006) provide overviews of the pertinent literature.

[9] See also, for example, Andrade and Le Bihan (2013), Fuhrer (2018), and Reslow (2019).

within a unified framework. We then use this framework to show how forecasters at the individual level, by contrast, overreact to the average information received between two periods.

Complementary to this paper, in contemporaneous and independent work, Bordalo *et al.* (2019) demonstrate similar overrevisions of individual-level forecasts to those that we document. In contrast to their paper, we show that these overrevisions mask evidence of both over- and underreactions to salient public information. We further show that such simultaneous over- and underreactions present a challenge for existing models of expectation formation, including Bordalo et al's (2019) theory of "diagnostic expectations".[10] Our theory overcomes this challenge, and is consistent with both over- and underreactions to new information. As Angeletos and Huo (2019) show, the approach proposed in this paper also has the advantage that the *as-if* myopia and anchoring that are important consequences of noisy rational expectations can be shown to directly carry over to our model of overconfidence.

Second, although forecaster information is sometimes acknowledged to be an upper bound of that held by the population at large (Marinovic *et al.*, 2013), most studies abstract from the particular characteristics that separate professional forecasters from the rest of the population. This has attracted criticism (e.g. Scharfstein and Stein, 1990 and Lamont, 2002), and given rise to a literature that looks at forecasters' incentives to distort their stated predictions (e.g. Laster *et al.*, 1999; Ehrbeck and Waldmann, 1996).[11] Our contribution in this context is to show, in a common framework, how several of the most prominent of such agency-based models are inconsistent with individual-level forecasts from a variety of professional surveys.

Third, our paper relates to the substantial body of work that links over- and underreaction of expectations to asset price anomalies. For example, Daniel *et al.* (1998) show how a model of overprecision (leading to overreactions) and self-attribution of skill (leading to underreactions) is consistent with the excess volatility and short-run momentum often found in financial markets.[12] Barberis *et al.* (1998), in contrast, show how a model of conservatism (underreaction) and "representativeness" (overreaction) can explain the underreaction of stock prices to earnings announcements jointly with the overreaction of stock prices to extreme events. Lastly, and closely related to our notion of relative overconfidence, Eyster *et al.* (2019) show how "cursedness" (the failure to extract information from market prices) may explain momentum in asset prices. In contrast to this work, our evidence of over- and underreactions is based directly on forecasters' stated predictions rather than the behavior of equilibrium objects, such as asset prices. We therefore view our evidence as a useful anchor for these models.[13]

[10] Closely related, Kohlhas and Walther (2018) and Ma *et al.* (2018) demonstrate how forecasters simultaneously extrapolate from recent events (i.e. overreact to past outcomes of the forecasted variable) but underreact to new information on average. Kohlhas and Walther (2018) show that a rational model of information choice is consistent with these stylized facts.

[11] See also, for example, Graham (1999), Laster *et al.* (1999), and Ottaviani and Sørensen (2006).

[12] Chan (2003) provides a review of the pertinent literature.

[13] Our results also connect to experimental work by Sunder (1992) and Landier *et al.* (2017), who, like us,

Organization: Section 2 presents a simple model of noisy rational expectations with mean-squared error preferences, and uses it to derive our main empirical predictions. Section 3 uses survey data to test these, while Section 4 shows how many prominent models of forecaster behavior do not explain the observed patterns that we find in the data. Section 5 therefore presents an alternative model, based on absolute and relative overconfidence, while Section 6 assesses its quantitative potential to match our empirical results. We conclude in Section 7. An appendix contains all proofs, while an online appendix contains further empirical results.

2 A Baseline Model

We start with a simple model of noisy rational expectations with mean-squared error preferences. We use this framework as a benchmark, both for our empirical results and for the alternative models considered later in the paper.

2.1 Model Setup

The model is comprised of a continuum of measure one of forecasters, indexed by $i \in [0, 1]$. Forecasters minimize the mean-squared error of their forecasts f_i of the random variable θ. All forecasters have the prior belief that $\theta \sim \mathcal{N}(\mu_i, \tau_\theta^{-1})$ and observe two types of information. Their own private information, summarized by the *private signal*

$$x_i = \theta + \epsilon_i, \tag{2.1}$$

where the noise terms $\epsilon_i \sim \mathcal{N}(0, \tau_x^{-1})$ are independent of θ and $\mathbb{E}[\epsilon_i \epsilon_j] = 0$ for all $j \neq i$. The private signal of one forecaster is not observed by any other forecaster. In addition to their private information, all forecasters observe the realization of the *public signal*

$$y = \theta + \xi, \tag{2.2}$$

where $\xi \sim \mathcal{N}(0, \tau_y^{-1})$ is independent of θ and ϵ_i for all $i \in [0, 1]$. The signal y is public in the sense that its realization is common knowledge among all forecasters. Within this framework, we focus on three implications of noisy rational expectations.[14]

find evidence of both over- and underreactions to new information, but conclude that overreactions are a more pervasive feature. We propose a model that is consistent with such behavior.

[14] Apart from the implications discussed below, broad aspects of survey forecasts are clearly consistent with noisy rational expectations. First, survey forecasts are dispersed and differ across forecasters (Zarnowitz, 1985). Second, forecasts are often smoother, with lower volatility, than the variable that is being forecasted (Ottaviani and Sørensen, 2006). In fact, one of Muth's (1961) aims in proposing the rational expectations hypothesis was to explain these two stylized facts (p. 316 in Muth, 1961).

2.2 Average Forecasts (Implication 1)

The first implication pertains to the behavior of average forecasts, and follows Coibion and Gorodnichenko (2015). Consider the problem faced by forecaster i. Based on private and public information, her optimal estimate of θ is:[15]

$$f_i(\theta) = \mathbb{E}[\theta \mid \mu_i, x_i, y] = \mu_i + k(\mathbb{E}[\theta \mid x_i, y] - \mu_i), \tag{2.3}$$

where $k \equiv \frac{\tau_x+\tau_y}{\tau_\theta+\tau_x+\tau_y}$ denotes the combined weight on new information (x_i and y). The presence of imperfect information implies that $k \in (0,1)$. The more precise a forecaster's information is, the closer k is to one. Averaging (2.3) across all i and rearranging, we arrive at

$$\theta - f(\theta) = a + b(f(\theta) - \mu) + \nu, \tag{2.4}$$

where $a = 0$, $b \equiv \frac{1-k}{k}$, $f(\theta) \equiv \int_0^1 f_i(\theta)\,di$ denotes the average forecast in the population, $\mu \equiv \int_0^1 \mu_i di$ the average prior expectation, and $\nu \equiv \frac{\tau_y}{\tau_x+\tau_y}\xi$. We have also used that $\int_0^1 \epsilon_i di = 0$, since the noise in forecasters' private signals cancels on average.

In a regression of average forecast errors $\theta - f(\theta)$ on average forecast revisions $f(\theta) - \mu$, the estimate of the slope coefficient $b = \frac{1-k}{k}$ should therefore be positive and measure the extent of information frictions ($k < 1$).[16] Forecasters revise their forecasts by less than a hypothetical individual would do if she could observe the average information in the population ($\int_0^1 x_i di = \theta$ and y). This is because forecasters downweigh their own information to account for its noisiness ($k < 1$). But since the private noise terms cancel on average, this downweighing of new information leads to a positive correlation between the average forecast error, on the one hand, and the average forecast revision, on the other hand ($b > 0$). Relative to the full information benchmark, noisy private information leads to an underrevision of the average forecast in response to the average information observed.

2.3 Individual Forecasts (Implication 2 and 3)

The second and third implication, by contrast, concern the behavior of individual forecasts. A well-known implication of conditional expectation forecasts is that individual forecast errors

[15] As discussed in e.g. Bhattacharya and Pfleiderer (1985) conditional expectations not only correspond to optimal predictors for mean-squared error loss functions but indeed for any symmetric loss function.

[16] Because of the presence of noise in public information, least-squares estimates of b in (2.4) are downwardly biased. Specifically, $\mathbb{E}[\theta - f\theta \mid f\theta - \mu] = \frac{1}{k}\left(1 - k - \frac{c^2\tau_\theta}{\tau_y+c^2\tau_\theta}\right)(f-\mu)$ from (2.3), where $\frac{1}{k}\left(1 - k - \frac{c^2\tau_\theta}{\tau_y+c^2\tau_\theta}\right) \in \left(0, \frac{1-k}{k}\right)$ for all $(\tau_\theta, \tau_x, \tau_y) \in \mathbb{R}^3_+$. As argued in Coibion and Gorodnichenko (2015), such downward bias, however, still entails that statistically significant findings of $b > 0$ imply average underreactions relative to full information since least-squares estimates will understate any positive association. Lastly, we note that if $\tau_x \to 0$, then $\frac{1}{k}\left(1 - k - \frac{c^2\tau_\theta}{\tau_y+c^2\tau_\theta}\right) \to 0$. Average underreactions are only due to private information.

are uncorrelated with linear combinations of the elements in forecasters' information sets.

Let z denote any linear combination of x_i, y, and μ_i. Then,

$$\mathbb{E}\left[\theta - f_i\theta \mid z\right] = \mathbb{E}\left[\theta \mid z\right] - \mathbb{E}\left[\mathbb{E}\left[\theta \mid x_i, y, \mu_i\right] \mid z\right] = 0, \tag{2.5}$$

since $\mathbb{E}\left[\mathbb{E}\left[\theta \mid x_i, y, \mu_i\right] \mid z\right] = \mathbb{E}\left[\theta \mid z\right]$ by the *Law of Iterated Expectations.*

Equation (2.5) has at least two important consequences. First, estimates of the slope coefficient in (2.4) at the individual level should equal zero. That is, estimates of β in

$$\theta - f_i\left(\theta\right) = \alpha + \beta\left(f_i\left(\theta\right) - \mu_i\right) + \nu_i, \tag{2.6}$$

should be indistinct from zero, in contrast to the regression at the average level in which the slope coefficient is positive ($b > 0$). Individual forecast errors $\theta - f_i\theta$ cannot be correlated with individual forecast revisions $f_i\theta - \mu_i$, themselves a combination of x_i, y, and μ_i. If they were, forecasters would be able to exploit this correlation to improve their forecasts, contradicting the assumption that their forecasts were the conditional expectation to start with.

Second, by a similar logic, (2.3) also implies that individual forecast errors should be uncorrelated with any public information y that forecast revisions are in part based on. As a result, the estimate of δ in the regression

$$\theta - f_i\left(\theta\right) = \alpha + \delta y + \nu_i. \tag{2.7}$$

should also equal zero, since any non-zero coefficient would contradict the assumption that forecasters correctly use the information contained in the public signal.

We summarize these three implications of noisy rational expectations in Proposition 1.

Proposition 1. *Individual forecast revisions and public information do not predict individual forecast errors ($\beta = 0$ and $\delta = 0$ in 2.6 and 2.7). Average forecast errors are, however, positively correlated with average forecast revisions ($b > 0$ in 2.4).*

Despite its simplicity, Proposition 1 nests several important cases. First, because of the potential for heterogenous priors $\mu_i \neq \mu_j$ for $i \neq j$, Proposition 1 covers the important case in which θ itself evolves dynamically across time, in accordance with for example an $AR(1)$. When new information is observed in each period, forecasts in (2.3) then correspond to those from the Kalman Filter (Anderson and Moore, 1979).[17]

[17] Furthermore, because the first two implications in Proposition 1 ($\beta = 0$ and $\delta = 0$) derive from the Law of Iterated Expectations, they extend to economies in which shocks are non-normal. In a dynamic context, the Law of Iterated Expectations also ensures that they extend to cases where the economy experiences structural breaks, as long as forecasters have a correct prior distribution about these. The first implication ($b > 0$)

Second, because of the presence of the private signal x_i in (2.1), Proposition 1 also extends to the popular case in which forecasters exhibit *rational inattention* (Sims, 1998, 2003). The optimal signal for a capacity constraint agent with entropy attention costs to observe is identical to x_i (Cover and Thomas, 2012). The only difference to Proposition 1 would be that δ would be positive in this case ($\delta > 0$ in 2.7). This is because, although individual forecasts still equal conditional expectations, f_i would be conditioned only on the optimal (private) signal $x_i^\star$ ($f_i = \mathbb{E}[\theta \mid \mu_i, x_i^\star]$).[18] We return to the rational inattention case in Section 4.

Finally, we note that a positive b in Proposition 1 corresponds to an average underrevision relative to the *full information rational expectations* benchmark. By contrast, a positive β or δ would reflect an underrevision and underreaction, respectively, relative to the *noisy rational expectations* case. We use both notions of over- and underrevisions interchangeably in what follows when there is no cause for confusion.

3 Empirical Evidence

In this section, we compare the implications of noisy rational expectations, listed in Proposition 1, to key features of US inflation forecasts. We document that professional forecasters' average inflation forecasts are in line with noisy rational expectations ($b > 0$). We then show that, at the individual level, the same forecasters by contrast make forecast revisions that are systematically too large ($\beta < 0$). Lastly, we document that these overrevisions at the individual level mask strong evidence of both over- and underreactions to salient public signals ($\delta < 0$ and $\delta > 0$). A robustness subsection shows that our results carry over to other forecast variables, alternative samples, and to other datasets. A section detailing the construction of the different data series is included in the Online Appendix.

3.1 Data

We focus on forecasts of US inflation from the *Survey of Professional Forecasters* (SPF).[19] At the start of each quarter, the SPF asks its respondents for their forecasts of a number of key macroeconomic and financial variables, and publishes them, in anonymous format

carries over to other affine prior-likelihood combinations, such as beta-binomial, gamma-poisson, and when observations are negative binomial, gamma, or exponential with natural conjugate priors (Ericson, 1969).

[18] Notice that $\mathbb{C}ov\left(\theta - f_i\theta,\, y\right) > 0$ because $\mathbb{V}[\theta] > \mathbb{C}ov\left(\theta, \mathbb{E}[\theta \mid x_i, \mu]\right)$. Thus, $\delta > 0$. However, it still the case that $b > 0$ and $\beta = 0$. Both follow from identical arguments to those in the main text.

[19] The SPF is the oldest quarterly survey of individual macroeconomic forecasts in the US, dating back to 1968. The SPF was initiated under the leadership of Arnold Zarnowitz at the American Statistical Association and the National Bureau of Economic Research, which is why it is also still occasionally referred to as the ASA-NBER Quarterly Economic Outlook Survey (Croushore, 1993).

but with personal identifiers, shortly thereafter.[20] We study SPF forecasts of the year-on-year percentage change in the GNP/GDP deflator, for which the survey includes consistent forecasts for the six quarters following the survey quarter. We focus on inflation forecasts for three reasons. First, because inflation expectations play a central role in the economy as determinants of wages, goods and asset prices. Second, to compare our estimates to those of previous studies, which have focused disproportionally on inflation. And third, because data on inflation forecasts are available for a substantially longer time-span than forecasts of other variables, such as output. Throughout, we consider first-release realizations of inflation to most accurately capture the precise definition of the variable being forecasted. Importantly for our purposes, although the precise schedule over the quarter has changed over time, the administrators of the SPF have consistently and publicly published the average of survey results well before sending out the next round of the questionnaire.[21] The information set of respondents therefore includes the consensus (or average) forecast from the previous quarter.

3.2 Average Forecasts

We first study the properties of average inflation forecasts. We denote individual i's forecast made in period t of the variable π in period $t+h$ as $f_{it}\pi_{t+h}$. We then calculate the average forecast as $f_t\pi_{t+h} = \frac{1}{N_t}\sum_{i=1,..N_t} f_{it}\pi_{t+h}$, where N_t denotes the number of forecasters in period t, and estimate the following regression equation:

$$\pi_{t+h} - f_t\pi_{t+h} = a + b\left(f_t\pi_{t+h} - f_{t-1}\pi_{t+h}\right) + \nu_t. \tag{3.1}$$

where $f_t\pi_{t+h} - f_{t-1}\pi_{t+h}$ denotes the revision in the average forecast between period $t-1$ and t. We thus estimate (2.4) when θ corresponds to inflation h periods ahead.

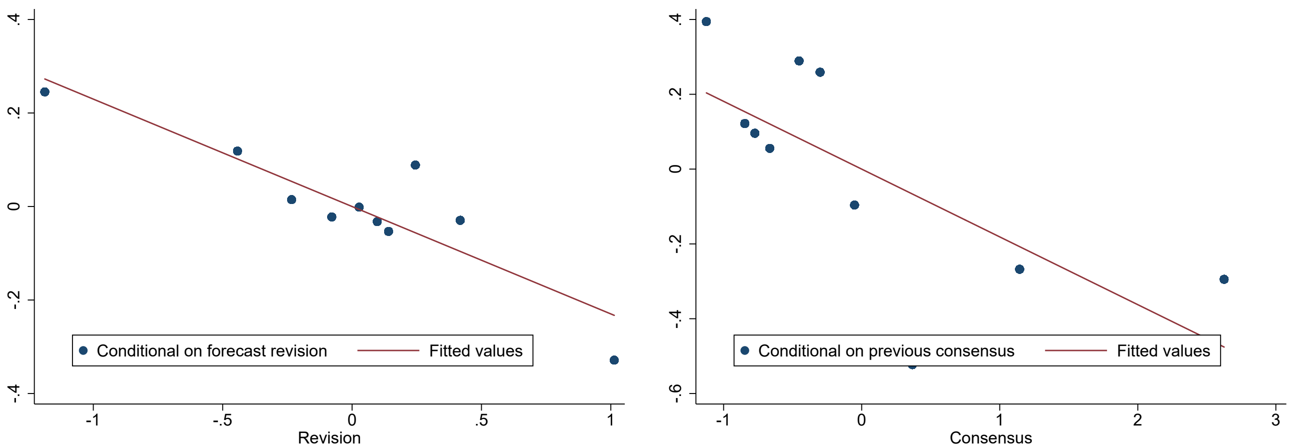
Table I presents the results for one-year ahead inflation forecasts ($h = 4$). Average forecast revisions are positively correlated with average forecast errors ($b > 0$). This effect is strong and highly significant, in line with the results in Coibion and Gorodnichenko (2015). Consistent with noisy rational expectations, forecasters on average underrevise their forecasts relative to the full information benchmark, leading to a positive correlation between average forecast errors, on the one hand, and average forecast revisions, on the other hand.[22]

[20]The number of respondents (professional forecasters from financial institutions, large industrial firms, and independent forecasting enterprises) fell from over 80 to around 20 by 1990. The Federal Reserve Bank of Philadelphia then took over the administration of the survey. The number of respondents has since fluctuated between 40 and 60. See https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-documentation.pdf?la=en for detail.

[21]See p.8 in the documentation: https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-documentation.pdf.

[22]Furthermore, consistent with the averaging out of noise in private information, average forecasts across our sample are also more precise than individual forecasts on average. This corroborates the empirical findings

Figure 1: Forecast Errors from the Survey of Professional Forecasters



The left-hand side panel depicts (on the vertical axis) the average of individual forecast errors taken within deciles of the distribution of individual forecast revisions (horizontal axis). The right-hand side panel shows (on the vertical axis) the average of individual forecast errors this time taken within deciles of the distribution of consensus forecasts from the previous wave of the SPF (horizontal axis). All variables are demeaned by subtracting their overall average during the SPF sample period (1971Q2-2016Q4).

Table I: Estimates from the Survey of Professional Forecasters

	Average Forecasts	*Individual Forecasts*		
	Forecast Error	*Forecast Error*	*Forecast Error*	*Forecast Error*
Forecast Revision	1.276***	-0.192***	–	-0.196***
	(0.274)	(0.0673)		(0.0683)
Previous Consensus	–	–	-0.189**	-0.191**
			(0.0809)	(0.0868)
Constant	-0.0570	0.0175	0.701**	0.656**
	(0.0735)	(0.0786)	(0.310)	(0.311)
Sample	71Q2-16Q4	71Q2-16Q4	71Q2-16Q4	71Q2-16Q4
Obs	184	5099	6814	5089
R^2	0.250	0.267	0.222	0.287

(i) Column one presents estimates of (3.1); column two and three estimates of (3.2) and (3.3).

(ii) White double-clustered standard errors (individual forecasts). * p<0.10, ** p<0.05, *** p<0.01.

(iii) HAC standard errors (average forecasts). * p<0.10, ** p<0.05, *** p<0.01.

3.3 Individual Forecasts

We now turn to our main empirical results, describing the statistical properties of individual inflation forecasts. Specifically, we test the implication of noisy rational expectations (with mean-squared error preferences) that forecast errors are orthogonal to any information, be it public or private, available to forecasters at the time of the forecast.

3.3.1 Overrevision of Individual Forecasts

The second implication of noisy rational expectations, listed in Proposition 1, is that individual forecast errors should be uncorrelated with individual forecast revisions ($\beta = 0$). Figure 1 shows that this implication is *prima facie* not borne out by the data. The conditional means of individual forecast errors are negatively associated with the means of individual forecast revisions (left panel), suggesting that β is negative. To test this implication more formally, we estimate a version of (2.6) at the individual level, using the benchmark specification:

$$\pi_{t+h} - f_{it}\pi_{t+h} = \alpha_i + \beta\left(f_{it}\pi_{t+h} - f_{it-1}\pi_{t+h}\right) + \nu_{it}, \tag{3.2}$$

where α_i denotes a forecaster-specific fixed effect. Equation (3.2) thus corresponds to (2.6) when θ is once more set equal to h-period ahead inflation π_{t+h}.

Table I confirms our initial impressions. The estimate of β is significantly negative and numerically large, inconsistent with Proposition 1. This negative estimated value of β implies that positive individual forecast revisions are associated with negative forecast errors. Forecasters on average revise their forecasts by too much relative to the noisy rational expectations benchmark, and hence on average overreact to the information received between subsequent survey rounds.

However, importantly, such overrevisions of individual forecasts do not inform us about the composition of responses that lead to $\beta < 0$. All we can conclude is that forecasters overreact *on average*. In particular, estimates of (3.2) do not allow us to separate between (i) whether the overrevision of forecasts is comprised exclusively of overreactions to new information, or (ii) whether the overall overrevision masks evidence of both over- and underreactions. As we argue in Section 4, this distinction is central to our analysis, as it will greatly constrain the set of models of expectation formation that are consistent with the data.

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of Zarnowitz and Lambros (1987).

3.3.2 Over- and Underreactions to Public Signals

To provide a first pass at a breakdown of individual forecasters' responses to new information, our second test at the individual level estimates a version of (2.7):

$$\pi_{t+h} - f_{it}\pi_{t+h} = \alpha_i + \delta y_t + \nu_{it}. \tag{3.3}$$

The third implication of noisy rational expectations, listed in Proposition 1, is that δ should equal zero for any public signal y.

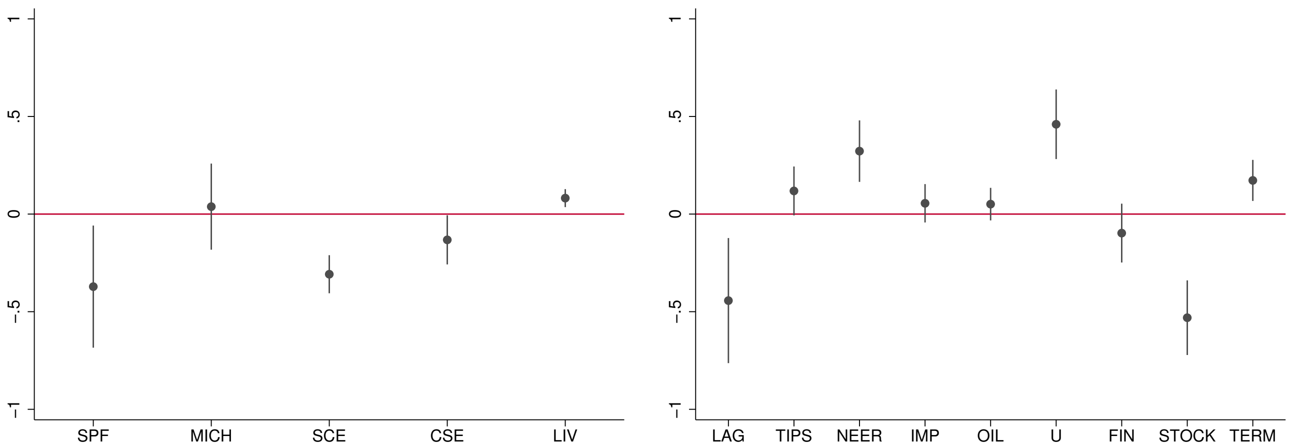
To estimate (3.3) requires a particular piece of public information that is at the same time publicly observed, relevant, and salient to professional forecasters. We first focus on a natural example of such public information within our context: that of the consensus forecast from the previous wave of the survey ($y_t = f_{t-1}\pi_{t+h}$). As argued in the introduction, and more forcefully in Ottaviani and Sørensen (2006), professional forecasters pay close attention to realizations of consensus. This is to assess how well they perform relative to their immediate competitors. Consensus forecasts should therefore provide a conservative benchmark against which to test the orthogonality of individual forecast errors to public information.

Figure 1 (right panel) depicts the conditional mean of individual forecasts errors for one-year ahead inflation ($h = 4$), and shows that these decrease in previous period's consensus forecast. Table I confirms this impression formally. The estimate of δ in (3.3) is negative and statistically significant, inconsistent with Proposition 1. Individual forecast errors are on average more negative not only when individual forecast revisions are more positive, but also when the previous consensus forecast is higher. We conclude that forecasters appear to overreact to the information contained in consensus forecasts. These overreactions are corroborated in the final column of Table I, where we report the coefficient estimates from a multivariate regression that includes both individual forecast revisions and consensus.

The negative estimate of δ in Table I may suggest that forecasters overreact to all new information observed (implying $\delta < 0$ for all public signals). However, Figure 2 shows that such uniformity does not exist. It presents estimates of δ from (3.3) using a variety of public signals. We divide this evidence into two types: (i) alternative survey measures of future inflation, similar to consensus forecasts (left-hand side panel), and (ii) other publicly observable time-series that are often used to predict inflation (right-hand side panel). We take the latter set of variables from the European Central Bank's published list of "important inflation indicators", to tie our hands with respect to variable selection.[23] A similar set of variables are used in Cecchetti (1995), Canova (2007), and Stock and Watson (2008), among many others. To make

[23] See, for example, https://www.ecb.europa.eu/pub/pdf/other/ebart201704_01.en.pdf. The main difference is that we avoid the inclusion of measures of the "output gap", since that would entail taking a stance the structural determinants of any deviations from the first-best allocation.

Figure 2: Inflation Forecast Errors and Different Public Signals



The figure depicts estimates of δ in (3.3) (on the vertical axis) for various public signals (on the horizontal axis). The left-hand side panel shows the coefficient estimates for previous period's consensus estimate of one-year ahead inflation from the Survey of Professional Forecasters (SPF), the Michigan Survey of Consumers (MICH), the Survey of Consumer Expectations (SCE), the Consensus Economics Survey (CSE), and the Livingstone Survey (LIV). The right-hand side panel, by contrast, shows estimates of δ using one-period lagged inflation outcomes (LAG), 10-year inflation expectations from the TIPS market (TIPS), the year-over-year change in the nominal effective exchange rate (NEER), the year-over-year change in import prices (IMP), the year-over-year change in the WTI oil price (OIL), the unemployment rate (U), the Cleveland Fed's Financial Market-based measure of future inflation (FIN), the year-over-year change in the SP500 (STOCK), and the 10-year-2-year term spread (TERM). All variables have been standardized, and have been signed such that an increase predicts higher inflation one year out. All variables and growth rates have also been derived using the latest available data at the time of the inflation forecast. Whisker-intervals correspond to 95 percent White doubled-clustered confidence bounds. Finally, note that the TIPS and the SCE are only available after 2015. See also the Online Appendix, which includes sample definitions for all variables.

our estimates comparable across series, all variables have been standardized, and have been signed such that an increase predicts higher inflation one year out.

On balance, we find that, although forecasters overreact to previous consensus forecasts from the SPF, the evidence for other public signals is substantially more mixed. For example, the left-hand side panel in Figure 2 shows that forecasters *overreact* with similar strength to the observation of professional forecasters' consensus estimate from *Consensus Economics*.[24] This reflects the fact that Consensus Economics covers many of the same forecasters as the SPF. However, forecasters *underreact* to the information contained in the consensus outcome from the much broader set of forecasters covered by the *Livingstone Survey* (Croushore, 1997), in addition to estimates of consumer expectations from the *Michigan Survey of Consumers* (Dominitz and Manski, 2003), although the latter is not statistically significant.[25]

The right-hand side panel in Figure 2 confirms this picture of over- and underreactions in response to public signals other than measures of average expectations. When we estimate the relationship between individual inflation forecast errors and nine common public signals of future inflation, we find significant overreactions to some (e.g. lagged outcomes, skin to extrapolation), but significant underreactions to others (e.g. financial market expectations of future inflation or changes to the unemployment rate). A simple ANOVA exercise shows that the probability of all the coefficients in Figure 2 occurring by chance in the absence of underlying over- or underreactions is less than 0.0001.

In Section 4, we show that such simultaneous over- and underreactions when combined with our previous results are inconsistent with a broad class of rational and behavioral models of expectation formation. However, before turning to this point, the next subsection shows that our results extend beyond the specific context of inflation forecasts from the SPF.

3.4 Robustness

The patterns documented in the SPF are remarkably stable when we consider other macroeconomic variables than inflation, alternative forecast periods and forecast horizons, as well as other forecaster surveys from the US and the Euro Area. Here, we summarize these results (Table II and Figure 3), presented in detail in the Online Appendix.

Under- and Overrevisions of Forecasts ($b > 0$ and $\beta < 0$): First, to complement our benchmark

[24] Consensus Economics is a monthly survey of professional forecasters' macroeconomic expectations. The survey was started in 1989 and covers several countries besides the United States.

[25] The Livingstone Survey is a bi-annual survey of forecaster expectations that covers many different types of forecasters. It is the oldest continuous survey of forecaster's expectations. The Federal Reserve Bank of Philadelphia took responsibility for the survey in 1990. The Michigan Consumer Survey is a monthly survey of a large number of U.S. consumers. It is based on a telephone survey that gathers information on consumer expectations regarding the overall economy.

Table II: Robustness and Alternative Estimates

Description	*Avr. Forecast Error*		*Ind. Forecast Error*			
	b-coef	Std. error	β-coef	Std. error	δ-coef	Std. error
GDP Deflator (SPF)	1.276	(0.274)	-0.192	(0.067)	-0.189	(0.081)
CPI Inflation (SPF)	0.270	(0.250)	-0.294	(0.097)	-0.439	(0.070)
Real GDP (SPF)	0.612	(0.245)	-0.193	(0.060)	0.209	(0.163)
GDP Deflator (SPF, post '92)	0.581	(0.228)	-0.372	(0.052)	-0.375	(0.101)
CPI Inflation (SPF, post '92)	0.210	(0.439)	-0.274	(0.174)	-0.518	(0.157)
Real GDP (SPF, post '92)	0.494	(0.346)	-0.107	(0.137)	-0.504	(0.237)
GDP Deflator (SPF, h=2)	0.617	(0.146)	-0.287	(0.051)	-0.058	(0.080)
GDP Deflator (SPF, finan.)	0.364	(0.223)	-0.366	(0.045)	-0.300	(0.116)
GDP Deflator (SPF, non-finan.)	0.621	(0.266)	-0.367	(0.060)	-0.316	(0.092)
HICP Inflation (EASPF)	0.782	(0.400)	-0.067	(0.154)	**-0.535**	(0.687)
Real GDP (EASPF)	0.638	(0.206)	0.367	(0.168)	-0.797	(0.178)
CPI Inflation (LIV)	-1.156	(0.754)	-0.518	(0.089)	-0.316	(0.114)
Real GDP (LIV)	0.826	(0.438)	-0.113	(0.147)	-1.215	(0.393)

The table shows estimates of b in (3.3), β in (3.2), and δ in (3.3), in which the estimates of δ use the previous period's consensus outcome from the survey in question. LIV denotes the Livingstone Survey, while EASPF refers to the Euro Area Survey of Professional Forecasters. All estimates are computed using year-on-year growth rates that have been derived using the latest available data at the time of the forecast (see Online Appendix, which includes sample definitions). Colored coefficients are significant at the five percent level. White doubled-clustered standard errors are used for forecasts at the individual level; HAC standard errors for average forecasts. Bold indicates a coefficient in which fewer than 50 time-clusters have been estimated, and that are significant using the adjustment in Cameron *et al.* (2010).

results using GNP/GDP inflation forecasts, we also consider forecasts of an alternative inflation measure (CPI) and real output growth (GDP) from the *Survey of Professional Forecasters* (Table II). The estimated coefficients for b and β all have the same sign as our benchmark results, and are all statistically significant with the exception of the CPI estimate of β. Similar results hold when we restrict the sample to after 1992, when the Federal Reserve Bank of Philadelphia took over the administration of the SPF and substantially increased its coverage (and when inflation was also lower and more stable than during the 1970s and 1980s).[26] We also document that similar results hold at a semi-annual forecast horizon ($h = 2$).

Second, we extend beyond the United States and consider professional forecasts of inflation for another geographic area, the Euro Area, as collected by the *ECB's Survey of Professional Forecasters* (Garcia, 2003). We once more find estimates of b of the same sign and magnitude to those from the US SPF. While the point estimates of β are positive, the uncertainty around these estimates is large because of the short estimation sample that starts only in 2000. As we discuss below, our model in Section 5 can in any case also account for such results.

Third, a large share of forecasters in the US and Euro Area SPF comes from financial sector institutions. We therefore also consider whether our results extend beyond financial sector forecasters. Table II shows that our results ($b > 0$ and $\beta < 0$) carry over with equal force to the non-financial sector forecasters in the US SPF, as well as to forecasters that are part of the *Livingstone Survey*. The non-financial sector forecasters in the US SPF mainly come from large private sector firms and consultancies, while the Livingstone Survey covers a broader range of non-financial sector institutions (these include academic institutions and government entities, for example). Online Appendix B.1 (Table 8) shows that our results also extend to the five different classifications of forecasters in the Livingstone Survey. Lastly, the Livingstone Survey estimates show that our results extend to semi-annual forecast revisions.

Over- and Underreactions to Public Signals ($\delta \lesseqgtr 0$): To further complement our baseline results, Table II and Figure 3 summarize various estimates of the under-/overreaction coefficient δ in (3.3), using different forecaster surveys and public signals than those considered above.

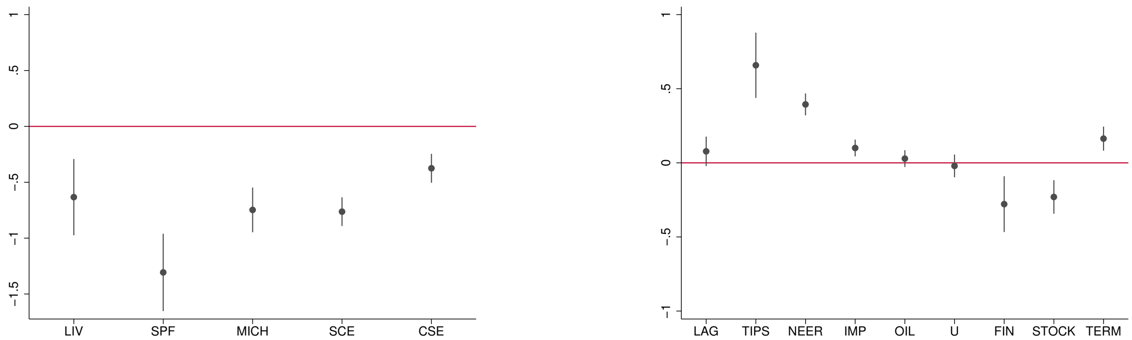
Consistent with our earlier results, Table II shows that forecasters overreact to previous period's consensus outcome from their own survey ($\delta < 0$). Furthermore, the estimates are of similar size to those reported in Table II. On balance, part of the overrevision of individual forecasts ($\beta < 0$) thus seems to arise because forecasters overreact to peer outcomes.

More generally, however, the overrevision of individual forecasts appear to be the product of both over- and underreactions to public information ($\delta \lesseqgtr 0$). Figure 3 confirms this picture of over- and underreaction, also visible in Figure 2. Similar to the baseline forecasts from the SPF,

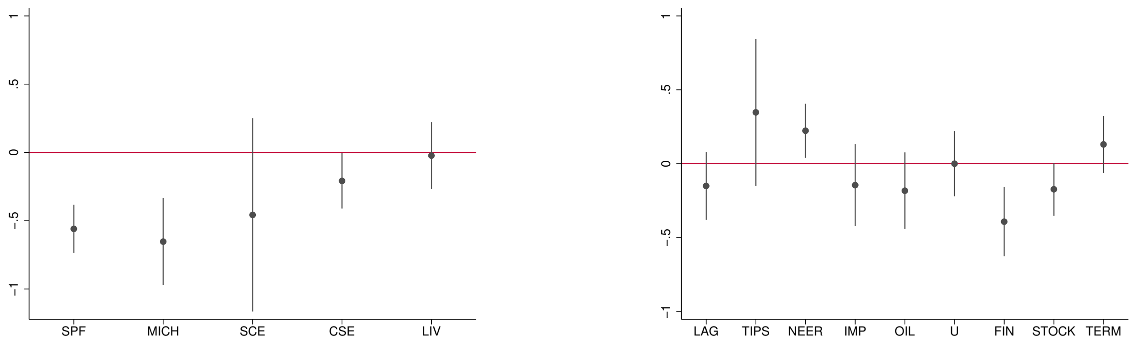
[26] The 1992Q1 observation corresponds to the first realization of five quarter-ahead inflation forecasts ($h+1$) from the SPF after the Federal Reserve Bank of Philadelphia took over the administration of the survey.

Figure 3: Robustness and Different Public Signals

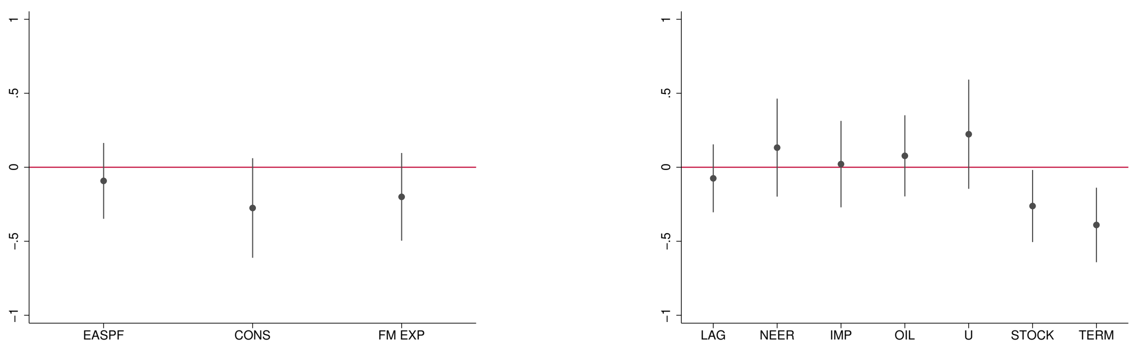
(a) Livingstone Survey: Inflation



(b) SPF: Alternative Inflation Measure (CPI)



(c) Euro Area Survey of Professional Forecasters: Inflation†



The figure shows estimates of δ in (3.3) (on the vertical axis) for various public signals (on the horizontal axis). The description of the different public signals used for the SPF and the Livingstone Survey is explained in the label for Figure 2. EASPF denotes the previous period's consensus forecast from the ECB's Survey of Professional Forecasters, CONS consumers' one-year ahead inflation expectations from the European Commission's Consumer Survey, and lastly FM EXP financial market expectation of one-year ahead inflation as derived from Euro Area inflation swaps. All variables have been standardized, and signed so that an increase predicts higher inflation one-year out. All variables and growth rates have also been derived using the latest available data at the time of the forecast. Whiskers correspond to 95 percent White double-clustered confidence bounds. † denotes coefficients that are estimated with fewer than 50 time-clusters. The recommended adjustment in Cameron *et al.* (2010) makes all coefficients (in Panel c) significant at the five percent level, with the exception of import and oil prices. See also the Online Appendix, which includes sample definitions.

the broader set of forecasters from the Livingstone Survey (Panel a) and the ECB's Survey of Professional Forecasters (Panel c) underestimate the inflationary effects one-year out of several publicly observable variables (such as movements in the exchange rate). However, forecasters equally overreact to the predictive power of others variables (such as stock price changes). The estimates from the ECB's Survey of Professional Forecasters are once again more uncertain than those from other surveys because of the short estimation sample. Adjusting the double-clustered standard errors for the short time dimension (as in Cameron *et al.*, 2010) makes all coefficients in Panel (c) significant at the five-percent level, with the exception of changes in import and oil prices. Panel (b) further shows that this coincidence of over- and underreactions carries over to SPF forecasts of an alternative inflation measure.

Combined, the above results (Table I, II, and Figure 2, 3) create a robust picture of response-coefficients δ in (3.3) that are both positive and negative, depending on the precise public signal in question. Sections 5 and 6 provide a model to shed light on the conditions under which such coincidence of over- or underreactions can arise.

Outliers, Important Individuals, and Noisy Forecasts: Finally, we consider the extent to which our estimates depend on select observations or individuals. Appendix D shows that our main results extend equally to case where we winsorize left- and right-hand side observations in (3.1), (3.2), and (3.3) at the one percent level. This appendix also shows that if drop forecaster i from the SPF consensus in the individual-level regression (3.3), the overreaction to consensus documented in Table I remains. Lastly, Section 4 shows that our main conclusions also do not depend on the assumption that survey forecasts are observed without noise.

3.5 Summary of Empirical Results

Taken together, our results suggest that *average* forecasts are consistent with models of noisy rational expectations with mean-squared error preferences ($b > 0$). This confirms the results of Coibion and Gorodnichenko (2015). *Individual* forecasts, however, show patterns that strongly contradict such models. Specifically, forecasters systematically overreact on average to the news that they receive between survey rounds. This leads to too large forecast revisions relative to the noisy rational expectations benchmark ($\beta < 0$). Consistent with this pattern of overall overrevisions, we find strong evidence of overreactions to a specific public signal that is salient in the context of professional forecasts, namely the consensus forecast from the previous round of the survey ($\delta < 0$). We, nevertheless, also find evidence of sizable underreactions to other public signals ($\delta > 0$). As we have argued in the introduction, and will show formally below, several prominent models of forecaster behavior, both rational and behavioral, struggle to explain this coincidence of over- and underreactive forecasts. The next section makes this explicit using the basic framework from Section 2.

4 A Common Framework for Alternative Models

At a general level, there are two classes of potential explanations for the observed predictability of individual forecast errors. First, agency-based explanations in which the assumption of mean-squared error loss is replaced, so that optimal forecasts do not correspond to conditional expectations. And second, behavioral explanations that keep the assumption of mean-squared error loss but allow for non-rational uses of information. In this section, we show that, although a variety of models are consistent with under- and overrevision of forecasts at the average ($b > 0$) and individual level ($\beta < 0$), respectively, neither can simultaneously account for both over- and underreaction to public information ($\delta \lesseqgtr 0$).

4.1 A More General Framework

We start by generalizing the framework from Section 2 in a direction that captures several alternative models. For brevity, we do not discuss models that are *prima facie* inconsistent with revisions in individual and average forecasts. Specifically, we first study the case in which forecasters attach arbitrary weight to private information. We then extend our results to the case in which forecasters also "misuse" public information (relative to mean-squared optimal).

Consider once more forecaster i's estimate of θ from Section 2, $f_i(\theta) = \mathbb{E}\left[\theta \mid \mu_i, x_i, y\right]$. We can alternatively write this forecast as

$$f_i\left(\theta\right) = \left(1 - k_x\right)\mathbb{E}[\theta \mid \mu_i, y] + k_x x_i, \tag{4.1}$$

where k_x denotes an individual forecaster's weight on private information. But suppose now that the actual weight that forecasters attribute to private information k_x differs from its mean-squared optimal value, which we denote by $k_{x,\star} \equiv \frac{\tau_x}{\tau_\theta + \tau_x + \tau_y}$ ($k_x \neq k_{x,\star}$). In that case, forecasters either over- or under-emphasize private information, depending on whether $k_x \gtreqless k_{x,\star}$. Such alternative use of private information can explain the overrevision of individual forecasts ($\beta < 0$) at the same time as the underrevision of average forecasts ($b > 0$).

To see this, consider first the coefficient on individual forecast revisions β in (2.6). A few simple and straightforward derivations show that[27]

$$\begin{aligned}\beta &= \mathbb{C}ov\left[\theta - f_i\theta, f_i\theta - \mu_i\right]\mathbb{V}\left[f_i\theta - \mu_i\right]^{-1} \\ &= -k_x\left(k_x - k_{x,\star}\right)\mathbb{E}\left[x_i - \mathbb{E}[x_i \mid \mu_i, y]\right]^2\mathbb{V}\left[f_i\theta - \mu_i\right]^{-1},\end{aligned} \tag{4.2}$$

such that $\beta < 0$ whenever $k_x > k_{x.\star}$. When forecasters attach more weight to private information than optimal, forecasters will, on average, overreact to the information that they receive between subsequent periods. This, in turn, leads to a negative correlation between individual forecast errors $\theta - f\theta_i$ and individual forecast revisions $f_i\theta - \mu_i$, precisely as in the data.

Furthermore, it is straightforward to show that as long as $k_x < 1$ in (4.1), this negative correlation coincides with an underrevision of the average forecast ($b > 0$).[28] This is because forecasters with $k_x < 1$ still respond less to private information than the optimal reaction to the average private signal, $\int_0^1 x_i di = \theta$ (which in this case equals one).

However, while an increased weight on private information is consistent with our first two stylized facts, it nevertheless leads to neither an over- nor an underreaction to public information. In fact, when $k_x > k_{x,\star}$ (or $k_x \neq k_{x,\star}$), individual forecast errors remain uncorrelated with the public signal. Consider the forecast error that results from (4.1)

$$\theta - f_i(\theta) = \theta - k_x x_i - (1 - k_x)\,\mathbb{E}[\theta \mid \mu_i, y]. \tag{4.3}$$

Taking conditional expectations based upon the public signal y then shows that

$$\begin{aligned}\mathbb{E}\left[\theta - f_i\theta \mid y\right] &= \delta \times y \\ &= (1 - k_x)\left(\mathbb{E}\left[\theta \mid y\right] - \mathbb{E}\left\{\mathbb{E}\left[\theta \mid \mu_i, y\right] \mid y\right\}\right) = 0,\end{aligned} \tag{4.4}$$

where the last equality follows from the *Law of Iterated Expectations*. As a result, despite the erroneous use of private information, individual forecast errors remain uncorrelated with the public signal ($\delta = 0$). The reason is that a regression of individual forecast errors onto any public signal only considers whether that source of information is employed to minimize forecast errors. It does not consider more broadly whether all sources of information, in

[27] Let $f_{i,\star}\theta$ denote the rational forecast from Section 2. Then,

$$\begin{aligned}\mathbb{C}ov\left[\theta - f_i\theta, f_i\theta - \mu_i\right] &= \mathbb{C}ov\left[\theta - f_{i,\star}\theta + f_{i,\star}\theta - f_i\theta, f_i\theta - \mu_i\right] \\ &= \mathbb{C}ov\left[f_{i,\star}\theta - f_i\theta, f_i\theta\right] = \mathbb{C}ov\left[(w_\star - w)\left(x_i - \mathbb{E}[\theta \mid \mu_i, y]\right), \mathbb{E}[\theta \mid \mu_i, y] + w\left(x_i - \mathbb{E}[\theta \mid \mu_i, y]\right)\right] \\ &= -k_x\left(k_x - k_{x,\star}\right)\mathbb{E}\left[x_i - \mathbb{E}[x_i \mid \mu_i, y]\right]^2\end{aligned}$$

since $\mathbb{E}[\theta \mid \mu_i, y] = \mathbb{E}[x_i \mid \mu_i, y]$.

[28] See the common noise model in Appendix A of Coibion and Gorodnichenko (2015). In our model, the presence of the public signal y introduces a common noise component in individual forecasts.

general, are accurately employed. Although forecasters with an "excessive" weight on private information do not optimally use private information to minimize forecast errors, conditional on this misuse, they still utilize public information efficiently. That is why $\mathbb{E}[\theta \mid \mu_i, y]$ enters in (4.1). This, in turn, leads the population coefficient δ to equal zero.

Proposition 2. *Consider a forecast by individual $i \in [0,1]$ of the form*

$$f_i(\theta) = (1 - k_x)\,\mathbb{E}[\theta \mid \mu_i, y] + k_x x_i, \quad k_x \in (k_{x,\star}, 1)\,. \tag{4.5}$$

Then, $b > 0$ in (2.4), $\beta < 0$ in (2.6), but $\delta = 0$ in (2.7).

A natural extension of the results in Proposition 2 is one that simultaneously skews forecasters use of private *and* public information away from their mean-squared optimal values. A tractable and common approach to do so is the special case in which

$$f_i(\theta) = \mu_i + k\left(\mathbb{E}\left[\theta \mid x_i, y\right] - \mu_i\right), \quad k \neq k_\star, \tag{4.6}$$

where k denotes the combined weight on private and public information, which can differ from its mean-squared optimal value of $k_\star \equiv \frac{\tau_x + \tau_y}{\tau_\theta + \tau_x + \tau_y}$. When k exceeds its optimal value ($k > k_\star$), forecasters in (4.5) over-emphasize new information contained in private and public signals relative to their prior expectation. In this sense, forecasters with $k > k_\star$ over-emphasize all news that is characteristic of updates relative to prior beliefs. When forecasters overreact to all information, the resulting forecasts from (4.5) can also be consistent with the documented behavior of forecast revisions ($b > 0$, $\beta < 0$). This occurs when $k \in (k_\star, 1)$.[29] But, because forecasters overreact to *all* information when $k > k_\star$ such forecasts are also inconsistent with the evidence for underreactions to public information ($\delta > 0$).

Corollary 1. *Consider a forecast by individual $i \in [0,1]$ of the form*

$$f_i(\theta) = \mu_i + k\left(\mathbb{E}\left[\theta \mid x_i, y\right] - \mu_i\right), \quad k \in (k_\star, 1)\,. \tag{4.7}$$

Then, $b > 0$ in (2.4), $\beta < 0$ in (2.6), but $\delta < 0$ in (2.7).[30]

[29]When $k > 1$, forecasters react more to their own private information than a fictitious individual, who observes the average private signal ($\int_0^1 x_i di = \theta$). Hence, b would in this case be negative ($b < 0$).

[30]Similar steps to those in (4.2) show that:

$$\begin{aligned} \beta &= -\left(k - k_\star\right) k \mathbb{E}\left[\mathbb{E}[\theta \mid x_i, y] - \mu_i\right]^2 \mathbb{V}\left[f_i\theta - \mu_i\right]^{-1} \\ \delta &= -\left(k - k_\star\right) \mathbb{C}ov\left[\mathbb{E}[\theta \mid x_i, y] - \mu_i, y\right] \mathbb{V}\left[y\right]^{-1}. \end{aligned}$$

Corollary 1 follows immediately from these conditions.

4.2 Applications and Extensions

A variety of models of forecaster behavior fall within the classes described by Proposition 2 and Corollary 1. These models are thus consistent with an underrevision of forecasts at the average level (and thus with $b > 0$), an overrevision at the individual level ($\beta < 0$), but inconsistent with an over- and underreaction to public information (as evidenced by $\delta \lesseqgtr 0$).

4.2.1 Applications

1. Strategic Diversification: Laster *et al.* (1999), Ottaviani and Sørensen (2006), and Marinovic *et al.* (2013) describe the market for professional forecasters as a winner-takes-all competition, where only the most accurate forecast is rewarded with a fixed payoff that is split equally among all winners. As a consequence, the equilibrium distribution of forecasts becomes an important determinant of forecasters' stated predictions. In a symmetric equilibrium, all forecasters choose to over-emphasize private information and follow (4.5) with $k_x > k_{x,\star}$,[31] because of this component.[32] In line with Proposition 2, such forecasters therefore simultaneously under- and overrevise their forecasts ($b > 0$ and $\beta < 0$). But since public information does not diversify individual forecasts away from those of others, it is still the case that $\delta = 0$.

2. Reputational Considerations: In Ehrbeck and Waldmann (1996), forecasters are rewarded by clients according to their reputation, formalized by the perceived accuracy of their forecasts. Furthermore, one set of professional forecasters has access to more precise private information than another. As a result, the set of forecasters that receive less precise information overreact to their private information in an attempt to mimic their more informed competitors. Ehrbeck and Waldmann (1996) show that these less able forecasters in equilibrium follow (4.5), where $k_x > k_{x,\star}$, while their more informed competitors simply set $k_x = k_{x,\star}$.[33] Reputational considerations are thus, on average, simultaneously consistent with the documented underrevision of forecasts at the average level ($b > 0$), as well as the observed overrevision by the average forecaster at the individual level ($\beta < 0$). But since forecasters do not differ in their access to public information, conditional on private information, all forecasters still use

[31] To see why, consider an individual forecaster who sets $k_x = k_{x,\star}$. Increasing the weight on private information ($k_x > k_{x,\star}$) leaves the probability of winning the contest approximately unchanged (as the posterior is flat at the conditional expectation). But more weight on private information also (in expectation) strictly reduces the mass of other forecasters that makes the same forecast. In equilibrium, all forecasters therefore choose to set w such that $k_x \in (k_{x,\star}, 1)$.

[32] See, for example, Proposition 4 in Ottaviani and Sørensen (2006) and Proposition 1 and Corollary 1 in Marinovic *et al.* (2013). Unlike these models, Proposition 2 allows for individual-specific priors.

[33] See the results on p. 24 of Ehrbeck and Waldmann (1996). We extend their model to explicitly account for public information. We assume that in the second period all forecasters as well as clients observe the public signal y in (2.2). We summarize all first-period information in the individual-specific prior μ_i. With the exception of these modifications all details are as in Ehrbeck and Waldmann (1996).

public information efficiently ($\delta = 0$, see also Appendix A.1).[34]

3. Behavioral Overconfidence: A considerable literature in psychology has documented that people tend to over-emphasize their own information (e.g. Moore and Healy, 2008). As discussed in, for example, Daniel *et al.* (1998), and more recently in Bordalo *et al.* (2019), such inherent overconfidence could provide a basis for overreactions to new information. In our context, overconfident forecasters believe the precision of their private information to be higher than it actually is ($\tau'_x > \tau_x$). Their forecasts thus follow (4.5) where $k_x \in (k_{x,\star}, 1)$. However, as argued above, such overconfidence in private information would *in and of itself* not result in overreactions to public information, because it does not also entail an amended use of public information conditional on private information. We return to how a suitably adjusted notion of behavioral overconfidence can capture our stylized facts in Section 5.

4. Models of Generalized Overreactions: A candidate explanation for the overreaction to overall new information ($\beta < 0$) and consensus ($\delta < 0$) that we document in Section 3 are models of "*generalized overreactions*". This includes Bordalo *et al.* (2018)'s theory of "*diagnostic expectations*" and Evans and Honkapohja (2012)'s theory of "*excess Kalman Gain learning*". In the former case, forecasters overreact to all new information, because it is perceived to be diagnostic (or representative) of updates relative to prior information, while in the latter case forecasters instead overreact to increase their speed of learning. Within our framework, these models are captured by (4.7) in Corollary 1 in which $k \in (k_\star, 1)$. Compared to the baseline model, forecasts from such models overreact to both private *and* public information. Because of these overreactions, forecasts are simultaneously consistent with the documented behavior of forecast revisions ($b > 0$, $\beta < 0$), as well as with the observed overreaction to public consensus outcomes ($\delta < 0$). However, because these models imply that forecasters overreact to *all* public information, they are inconsistent with the evidence for underreactions to other public signals ($\delta > 0$), documented in Section 3 (Appendix A.2).

5. Underreactions and Rational Inattention: We close this list by noting that several other, prominent models of forecaster behavior (both rational or behavioral) fall within the classes described by Proposition 2 and Corollary 1 but where $k_x \in (0, k_{x,\star})$ or $k \in (0, k_\star)$. For example, Graham (1999), Lamont (2002), and Ottaviani and Sørensen (2006) describe models in which forecasters all have a rational incentive to herd, as in Scharfstein and Stein (1990).[35]

[34] In the spirit of Ehrbeck and Waldmann (1996), an alternative reputational model is one in which the interpretation of information is what separates forecasters. However, similar to Ehrbeck and Waldmann (1996), such a model would entail a negative relationship between the absolute size of individual forecast revisions and the absolute value of forecast errors. This is because forecasters who are better at interpreting information, and hence observe more precise signals, would revise their forecasts by more in equilibrium. As Appendix A.5 shows, such a relationship is inconsistent with the data.

[35] See also, for example, Croushore (1997), Welch (2000), and the literature review in Marinovic *et al.* (2013).

Ehrbeck and Waldmann (1996), by contrast, describe an alternative model of reputational concerns, in which able forecasters have more precise prior information, while Hirshleifer *et al.* (2011) detail a model in which security analysts for behavioral reasons underreact to their own private information. All of these explanations can be described within the classes described by Proposition 2 and Corollary 1, but where $k_x \in (0, k_{x,\star})$ or $k \in (0, k_\star)$. As a result, these models cannot explain the observed overrevision of individual forecasts ($\beta < 0$, Table I and II). Finally, we note that models of rational inattention (e.g. Sims, 2003) are likewise inconsistent with overrevisions ($\beta < 0$). As argued in Section 2, this is because rational inattention forecasts equal conditional expectations, and hence satisfy the Law of Iterated Expectations.[36]

4.2.2 Extensions

1. Rational Models with Strategic Complementarity and Error Correlation: We have discussed several models in which strategic incentives skew the optimal use of information away from its mean-squared optimal value. In place of these more specific models, a more general way to capture the basic idea that strategic incentives may skew individuals' use of private and public information is to extend the baseline framework to allow for arbitrary strategic complementarity between individual forecasts. Suppose forecaster i's estimate of θ follows

$$f_i(\theta) = (1-r)\mathbb{E}\left[\theta \mid \mu_i, x_i, y\right] + r\mathbb{E}\left[f(\theta) \mid \mu_i, x_i, y\right], \tag{4.8}$$

where $f(\theta) = \int_0^1 f_i(\theta)\, di$ and $r \in (-1, 1)$ denotes the amount of strategic complementarity (substitutability) between forecasters. The case where $r \to 1$ corresponds to the situation in which forecasters care only about aligning their forecasts to the average estimate in the population. By contrast, $r = 0$ corresponds to the benchmark case from Section 2. In addition, following Myatt and Wallace (2011), we also allow for arbitrary correlation between the errors in public and private information.[37] As shown by Angeletos and Pavan (2007), the coefficient r maps directly into the weight on private information k_x. Specifically, whenever there is strategic complementarity ($r > 0$), the weight on private information falls below its mean-squared optimal value ($k_x < k_{x,\star}$), and conversely when there is strategic substitutability ($r < 0$). Appendix A.3 shows that, despite the flexible amount of strategic complementarity and the presence of error correlation, individual forecast errors remain uncorrelated with

[36] Let $x_i^\star$ denote the optimal signal observed by a capacity constrained agent with entropy attention cost. Following Cover and Thomas (2012), $x_i^\star$ follows (2.1) but with a precision $\tau_x^\star \neq \tau_x$ that depends upon the capacity constraint. The rational inattention forecast equals $f_i(\theta) = \mathbb{E}\left[\theta \mid \mu_i, x_i^\star\right]$. But then the exact same steps as those taken in Section 2) show that $\beta = 0$, because of the Law of Iterated Expectations.

[37] Specifically, we allow for a common noise component: Forecasters' private information takes the form $x_i = \theta + \epsilon_i + cu$, where $c \in \mathbb{R}$ and $u \sim \mathcal{N}\left(0, \tau_u^{-1}\right)$, while the public signal y is $y = \theta + u + \xi$. The coefficient c controls the correlation between the error terms.

public information ($\delta = 0$). Indeed, a simple extension of (4.4) that takes into account the higher-order expectations that arise from (4.8) immediately establishes this result.

2. Trembling-hand Noise: Let $\tilde{f}_i(\theta) \equiv f_i(\theta) + e_i$ denote forecaster i's *stated* forecast, while $f_i(\theta)$ denotes her *actual* forecast. We further assume that $e_i \sim \mathcal{N}(0, \tau_e^{-1})$. In this case, forecasters stated predictions are subject to ("trembling-hand") noise. The results in Proposition 2 and Corollary 1 to a large extent carry over to this case. In fact, as Appendix A.4 shows, the only difference between such trembling-hand forecasts and those analyzed in this section is that the coefficient on individual forecast revisions becomes more negative. Let the slope coefficient from the individual-level regression in (2.6), using forecasters' noisy stated predictions, be denoted by $\tilde{\beta}$. Then, $\tilde{\beta} = \chi\left(\beta - \tau_e^{-1}\mathbb{V}\left[f_i\theta - \mu_i\right]^{-1}\right)$, where $\chi \equiv \frac{\tau_e}{\tau_e + \mathbb{V}[f_i\theta - \mu_i]^{-1}}$. As a consequence, even when forecasters are rational, and their actual forecast corresponds to their conditional expectation, forecasters still appear to overrevise their forecasts ($\tilde{\beta} = -\frac{\mathbb{V}[f_i\theta - \mu_i]^{-1}}{\tau_e + \mathbb{V}[f_i\theta - \mu_i]^{-1}} < 0$). However, importantly, our results on the correlation between individual forecast errors and public information remain as before. The coefficient δ equals that in Proposition 2 and Corollary 1 ($\tilde{\delta} = \delta$). In particular, it is still the case that conditional expectation forecasts remain uncorrelated with public information ($\tilde{\delta} = 0$). Finally, we note that for conditional expectation forecasts to be consistent with our estimate of β in, for example, Table I, the standard deviation of the noise should be around 40 percent of the standard deviation of the forecast revision. Although we doubt that forecasters' stated predictions are subject to this much noise (see Juodis and Kucinskas, 2019), the result suggests that our third implication in Proposition 1 ($\delta = 0$) provides a more robust test of of noisy rational expectations than the second ($\beta = 0$).

4.3 Towards a Theory of Misperceived Public Information

In this section, we have shown that several prominent models can explain the under- and overrevision of forecasts at the average ($b > 0$) and individual level ($\beta < 0$), respectively. However, neither of these models have been simultaneously consistent with the over- and underreaction to public information ($\delta \lesseqgtr 0$). At its core, there are two reasons for this result.

- First, when forecasters' follow a condition akin to (4.5), in which they only overemphasize private information, they still (conditionally) extract the correct information from public signals. The Bayes' conditional expectation $\mathbb{E}\left[\theta \mid \mu_i, y\right]$ enters immediately into (4.5). Proposition 2 and its extensions show that this, in turn, implies that the over- and underreaction coefficient δ always equals zero. Hence, to explain the survey data, forecasters necessarily have to misperceive the information content of public signals.

- Second, these misperceptions need to take a flexible form. As Corollary 1 and its extension shows, they cannot, for example, always result in an excessive weight placed on public information. Whatever misperception we consider needs to result in both too much as well as too little weight assigned to public information.

Proposition 3 summarizes and formalizes the latter requirement.

Proposition 3. *A forecaster's (linear) prediction $f_i(\theta) = k_\mu \mu_i + k_x x_i + k_y y$, in which $k_j \geq 0$ for $j = \{\mu, x, j\}$ and $\sum_{j=\{\mu,x,y\}} k_\mu = 1$ overreacts (underreacts) to public information if and only if $k_y > k_{y,\star}$ ($k_y < k_{y,\star}$), where $k_{y,\star}$ denotes the noisy rational expectation weight.*

The proposition shows the necessary and sufficient condition for forecasts to be consistent with the over- and underreactions to public information documented in the survey data. Specifically, whatever (linear) model of expectation formation that one considers, for it to be consistent with the survey data, forecasters have to be able to assign both *more* ($k_y \geq k_{y,\star}$) as well as *less* ($k_y \geq k_{y,\star}$) weight to public information than optimal.

A natural question this raises is what the source of such flexible misperceptions could be. As the next section shows, a natural candidate arises from forecasters' potentially incorrect views about other's information. This is especially the case when we consider public signals, such as consensus, that are simple averages of forecasters' stated predictions. The only possible source of misperception is, in this case, the beliefs about the forecasting rules that others follow, and in particular other forecasters' use and precision of information.

5 A Theory of Absolute and Relative Overconfidence

The previous section argued that forecasters have to flexibly misperceive public signals to match the observed over- and underreactions to public information ($\delta \lesseqgtr 0$) at the same time as the documented under- and overrevisions of forecasts ($b > 0$ and $\beta < 0$). In this section, we show how a simple model of overconfidence, combined with the fact that most public signals reflect others' choices, can account for all three stylized facts. The next section then explores the quantitative potential of our model to match our empirical estimates.

5.1 Overconfidence and Endogenous Public Information

Our model adds two key features to the basic framework from Section 2.

First, to capture a salient feature of the survey data, we relax one assumption of noisy rational expectations and allow forecasters to be overconfident in their information. In their overview of behavioral finance, De Bondt and Thaler (1985) state that "perhaps the most robust finding in the psychology of judgment is that people are overconfident" (p. 6). Specifically, we call overconfident those individuals that are not only overconfident in the precision of their own information but also wrongly think that their information is better than that of

Table III: Forecaster Overconfidence in the SPF

Estimation Method	*Confidence Interval*	
	95 percent	66 percent
SPF Density Implied	0.82***	0.56**
Giordani and Soderlind (2003)	0.72**	0.48**

The table shows the implied coverage ratio (the fraction of cases when actual inflation is inside a forecaster's confidence band). If forecasters were rational, a 95% confidence band would contain the true but unknown value 95% of the times. The confidence bands from the SPF are derived assuming a normal distribution and are calculated as: mean of inflation individual forecast $\pm$ critical value $\times$ standard deviation. Actual inflation is measured as the percentage change in the GDP Deflator (annual-average) in Q4 of each year. The significance of differences between the nominal confidence level and the actual are assessed using Christoffersen's (1998) test. * p<0.10, ** p<0.05, *** p<0.01. The sample is from 1981Q1 to 2018Q4. For reference, the table also includes the estimates computed using a similar method in Giordani and Söderlind (2003).

others. We therefore merge the two related but distinct notions of overconfidence commonly used in the psychology literature (Moore and Healy, 2008). We refer to the former type as *absolute overconfidence*, or *overprecision*, and the latter as *relative overconfidence*, or *overplacement* (Benoît *et al.*, 2015). Notice that it is the latter, relative aspect of overconfidence that differentiates the notion of overconfidence studied here from that explored in Section 4.

Evidence on absolute and relative overconfidence is abundant in the literature, and quantifiable using the same survey data that we have considered so far. Table III provides an example, using the individual-level density forecasts provided in the SPF (alongside the point forecasts). It shows that forecaster's stated precision of their one-year ahead inflation forecasts exceeds their actual precision by a sizable amount. The estimated *coverage ratio* of forecasters' 95 percent confidence interval, which describes the percentage of times when inflation outcomes fall inside an individual forecaster's confidence interval, is only between 72 and 82 percent, depending on the estimation method.

Other prominent examples of overconfidence include the stated precision of forecasts produced by financial market traders, the certainty in the diagnosis of severe illnesses by physicians, and the probability of a positive verdict by procedural lawyers.[38] Furthermore, as documented by Griffin and Tversky (1992) and others, overconfidence tends to be more prevalent for highly-qualified professionals, such as professional forecasters. It also tends to be more prevalent for more difficult tasks with a larger judgment component and delayed feedback (e.g. Einhorn, 1980, Moore and Dev, 2017). Section 6 further empirically explores forecasters' overconfidence, and shows that the estimates of absolute and relative overconfidence necessary to match our empirical results align with those from the survey data.

Second, we explicitly account for the fact that most public signals are *endogenous*. A

[38]See, for example, Oskamp (1965), Froot and Frankel (1989), Baumann *et al.* (1991), Benoît *et al.*, 2015, and the summaries in Odean (1998), Thaler (2000), and Moore and Healy (2008)

central feature of the information landscape that people observe is that most of it reflects the realized choices of others in the economy. This is true whether we consider data releases on past inflation or output, the observation of asset or goods prices, or the observation of previous period's consensus estimate. Because of this endogeneity of public information, any *equilibrium* model of expectation formation requires an assumption about people's views about the precision of others' information, such as that embedded in relative overconfidence. For example, the less precise people think others' private information is, the less informative they will think public signals that agglomerate such information are, and the less they will respond to them (e.g. Banerjee, 1992; Vives, 1997, Amador and Weill, 2010). This importance of people's views about others' information becomes especially salient when we consider public signals such as consensus forecasts, which reflect simple averages of individual expectations.

We now turn to how a simple model of absolute and relative overconfidence, when combined with the endogeneity of public information, maps into the conditions of Proposition 2 and 3. As a result, we show that overconfidence can account for all three stylized facts documented in Section 3. The next section shows that the model can also quantitatively match the survey evidence, and is consistent with moments of the data beyond those targeted.

5.2 A Two-Period Model of Overconfidence

Consider a two-period version of the baseline model from Section 2. We only introduce an explicit account of time in order to later equate the public signal that forecasters observe with the previous period's consensus estimate. At the start of each period, each forecaster $i \in [0, 1]$ receives the private signal x_{it} about the random variable $\theta \sim \mathcal{N}(\mu, \tau_\theta^{-1})$,

$$x_{it} = \theta + \epsilon_{it}, \quad t = \{1, 2\},$$

where $\epsilon_{it} \sim \mathcal{N}(0, \tau_x^{-1})$ is independent of θ and $\mathbb{E}[\epsilon_{it}\epsilon_{jh}] = 0$ for all $j \neq i$ and all $t, h = \{1, 2\}$. Unlike in the baseline model in Section 2, forecasters exhibit *absolute* and *relative overconfidence* in their private information. They believe the precision of their private signals to equal $\tau_x' > \tau_x$, and thus to be greater than the truth (absolute overconfidence). At the same time, forecasters also believe that other forecasters' private signals have a precision smaller than their own (relative overconfidence), and equal to $\hat{\tau}_x < \tau_x'$. We make no assumptions about the relative size of $\hat{\tau}_x$ and τ_x. At the start of the second period, each forecaster, in addition to private information, observes an *endogenous* public signal of the form:

$$y = \alpha_1\theta + \alpha_2 f(\theta) + \xi, \tag{5.1}$$

where $\alpha_j \geq 0$ for $j = \{1, 2\}$, $f(\theta) = \int_0^1 f_{i1}(\theta)\, di$ once more denotes forecasters' average expectation, and $\xi \sim \mathcal{N}\left(0, \tau_y^{-1}\right)$.[39] The key difference between the public signal in (5.1) and that explored in Section 2 is the endogeneity of the signal to individual actions, and thus expectations. For example, when $\alpha_1 = 0$ and $\alpha_2 = 1$, (5.1) directly becomes the consensus estimate from the previous period. Vives (2010) and Veldkamp (2011) summarize the importance of public signals of the form (5.1) for the social value of public information, the benefits of social learning, and the volatility of asset prices and business cycles, among others.

5.3 Individual Forecasts and Public Information

We proceed in two steps. We first derive individual forecasts in the first and second period, and show how relative overconfidence causes forecasters to flexibly misperceive public information. We then provide a set of sufficient conditions for second-period forecasts to be consistent with all three stylized facts documented in Section 3 ($b > 0$, $\beta < 0$, and $\delta \lesseqgtr 0$).

First-Period Forecasts and Public Information: Consider forecaster i's first-period forecast

$$f_{i1}(\theta) = v x_{i1} + (1 - v)\mu, \quad v \equiv \frac{\tau_x'}{\tau_x' + \tau_\theta}, \tag{5.2}$$

where v exceeds the mean-squared optimal weight on private information, $v_\star \equiv \frac{\tau_x}{\tau_x + \tau_\theta}$, because of forecasters' (absolute) overconfidence in their private information. Equally importantly, v also exceeds the weight that forecasters believe others place on their own private information (because of relative overconfidence), equal to $\hat{v} \equiv \frac{\hat{\tau}_x}{\tau_\theta + \hat{\tau}_x}$.

Let $\mu_i \equiv f_{i1}\theta$ denote forecaster i's prior expectation at the start of the second period with believed precision $\tau_\mu \equiv \tau_\theta + \tau_x'$.[40] To derive forecaster i's $t = 2$ forecast, we first need to differentiate between two different public signals: (i) the *realized public signal* y, and (ii) the *suspected public signal* $\hat{y}$. The former measures the *actual* noisy signal in (5.1),

$$y = \alpha_0 \theta + \alpha_1 \int_0^1 f_{i1}(\theta)\, di + \xi = \eta\theta + \xi, \tag{5.3}$$

where $\eta \equiv (\alpha_0 + \alpha_1 v)$ and we for simplicity abstract from the (constant) common prior. The latter, by contrast, measures the public signal that forecasters *believe* they observe,

$$\hat{y} = \hat{\eta}\theta + \xi, \tag{5.4}$$

[39] We restrict the sign of the $\alpha-$coefficients to avoid having to always separate between positive and negative signals of the fundamental in our discussion below. Neither of our main results depend critically upon this assumption. The reason for the introduction of the shock ξ in (5.1) is further purely technical. The important role it plays is to limit forecasters' ability to infer the true value of θ from the observation of y. The use of "non-invertibility" shocks has been standard in the noisy rational expectations literature since Hellwig (1980).

[40] From (5.2), this prior is equivalent to the observation of the signal $\mu_i = \theta + e_i$, $e_i \sim \mathcal{N}\left(0, \tau_\mu^{-1}\right)$.

where $\hat{\eta} \equiv (\alpha_0 + \alpha_1 \hat{v})$. Notice that the signals y and $\hat{y}$ differ only due to forecasters' misperceptions about the overconfidence of others ($\eta > \hat{\eta}$); that is, because all forecasters attach a weight of $v > \hat{v}$ to private information in (5.2). Relative overconfidence boils down to a simple one-parameter deviation from noisy rational expectations.

Misperceptions about Public Information: There are two important differences between (5.3) and (5.4), both of which are caused by forecasters' relative overconfidence ($\eta > \hat{\eta}$).

First, the realized public signal in (5.3) is more precise than the suspected one in (5.4). The precision of y about θ is $\eta^2 \tau_y$, while the precision of $\hat{y}$ is only $\hat{\eta}^2 \tau_y$, where $\eta^2 \tau_y > \hat{\eta}^2 \tau_y$ since $v > \hat{v}$.[41] Since all forecasters respond more to their own private information than expected by others, the public signal embeds more of the truly new (private) information that forecasters can learn from one another. This, in turn, increases the informativeness of the public signal above what forecasters believe to be true.

Second, and related, overconfident forecasters also over-infer movements in the fundamental from the public signal. The realized public signal loads onto the fundamental with η in (5.3), while the suspected consensus only loads onto the fundamental with $\hat{\eta} \in (0, \eta)$ in (5.4). Thus, a movement of $d\theta > 0$ in the fundamental causes forecasters to, all else equal, believe in a movement equal to $(\eta/\hat{\eta})d\theta > d\theta$, based on the observation of the public signal alone.

Put succinctly, the misperceptions about others inherent to our notion of relative overconfidence cause forecasters to simultaneously underestimate the precision of the public signal and to over-infer movements in the fundamental from it. Indeed, it is precisely because of these forces that forecasters will be able to both over- or underreact to its realization.

Second-period Forecasts: We are now ready to state forecasters' second-period forecast:

$$f_{i2}(\theta) = (1 - k_x)\,\hat{\mathbb{E}}[\theta \mid \mu_i, \hat{y}] + k_x x_{i2} \equiv k_\mu \mu + k_y y + k_x x_{i2} \tag{5.5}$$

where $\hat{\mathbb{E}}[\theta \mid \mu_i, \hat{y}] \neq \mathbb{E}[\theta \mid \mu_i, y]$ denotes forecaster i's conditional expectation of θ based on μ_i and the realized public signal y being treated as if it were $\hat{y}$.[42] We note that $k_x \in (k_{x,\star}, 1)$ and $k_y \lesseqgtr k_{y,\star}$, where $k_{x,\star}$ and $k_{y,\star}$ denote the mean-squared optimal weight on private and public information, respectively. As a result, we conclude that forecasters' second-period prediction map into the conditions stated in Proposition 3.

[41] Notice that the observation of y in (5.3) is proportional to the observation of $\theta + \frac{1}{\eta}\xi$. It follows that $\mathbb{V}\text{ar}[y \mid \theta] = \eta^{-2}\tau_y^{-1}$. Thus, the precision of y about θ is $\eta^2 \tau_y$.

[42] The weights k_x and k_y equal: $k_x = \frac{\tau_x'}{\tau_\theta + 2\tau_x' + (\alpha_0 + \alpha_1 \hat{v})^2 \tau_y}$, and $k_y = \frac{(\alpha_0 + \alpha_1 \hat{v})^2 \tau_y}{\tau_\theta + 2\tau_x' + (\alpha_0 + \alpha_1 \hat{v})^2 \tau_y}$.

5.4 Over- and Underreactions to Public Information

Because of the implication that $k_y \lesseqgtr k_y^{\star}$, a correlation naturally arises between individual forecast errors, on the one hand, and the public signal, on the other hand. Taking conditional expectations of i's second-period forecast error based upon the *realized* public signal y shows:

$$\begin{aligned} \delta \times y = \mathbb{E}\left[\theta - f_{i2}\theta \mid y\right] &= (1-k_x)\left(\mathbb{E}\left[\theta \mid y\right] - \mathbb{E}\left[\hat{\mathbb{E}}\left[\theta \mid \mu_i, \hat{y}\right] \mid y\right]\right) \\ &= (1-k_x)\,\mathbb{E}\left\{\mathbb{E}\left[\theta \mid \mu_i, y\right] - \hat{\mathbb{E}}\left[\theta \mid \mu_i, \hat{y}\right] \mid y\right\} \neq 0. \end{aligned} \tag{5.6}$$

Unlike in (4.4), the Law of Iterated Expectations in (5.6) no longer implies orthogonality between individual forecast errors and public information. This is because $\mathbb{E}\left\{\hat{\mathbb{E}}\left[\theta \mid \mu_i, \hat{y}\right] \mid y\right\}$ $\neq \mathbb{E}\left[\theta \mid y\right]$ when $\hat{y} \neq y$. The misperception of the public signal breaks the implication of the Law of Iterated Expectations that forecast errors are orthogonal to public information.

Importantly, (5.6) allows us to also directly characterize the forces that determine whether individuals over- or underreact to the public signal ($\delta \lessgtr 0$).

Proposition 4. *The correlation δ in (5.6) between individual forecast errors $\theta - f_{i2}(\theta)$ and the public signal y is given by*

$$\delta = \Delta(\kappa - \hat{\kappa}), \tag{5.7}$$

where $\Delta \in \mathbb{R}_+$, $\kappa \equiv \frac{\eta^2\tau_y}{\tau_\mu + \eta^2\tau_y} \times \frac{\hat{\eta}}{\eta}$ denotes the rational weight on the public signal y in $\mathbb{E}\left[\theta \mid \mu_i, y\right]$, while $\hat{\kappa} \equiv \frac{\hat{\eta}^2\tau_y}{\tau_\mu + \hat{\eta}^2\tau_y} \times 1$ denotes the misperceived weight in $\hat{\mathbb{E}}\left[\theta \mid \mu_i, \hat{y}\right]$.

Intuitively, how forecasters respond to endogenous public information, such as past consensus outcomes, depends on their views about its precision and its interpretation. As argued above, relative overconfidence causes forecasters to mistake both. On the one hand, it causes forecasters to underestimate the precision of public signals. The realized public signal y is more precise than the suspected public signal $\hat{y}$ ($\eta^2\tau_y > \hat{\eta}^2\tau_y$). This dismissal of other forecasters' information straightforwardly leads individuals' to *underreact* to public information ($\delta > 0$ because it causes $\kappa > \hat{\kappa}$). On the other hand, overconfidence also causes forecasters to over-infer movements in fundamentals from public signals. The realized public signal has a larger loading on the fundamental than the suspected one ($\eta/\hat{\eta} < 1$). This misinterpreation of the public signal, in turn, leads forecasters to *overreact* to its realizations. When forecasters over-infer values of the fundamental from observations of the public signal, they all else equal attach more weight to it than warranted ($\delta < 0$ since it causes $\kappa < \hat{\kappa}$).

In effect, (5.7) shows that both *under*- but also *overreactions* to public information can arise from individuals' dismissal of other's private information. This contrasts our results with those of Eyster *et al.* (2019), in which only the former can occur in equilibrium. Indeed, using

(5.7), the condition for $\delta \lesseqgtr 0$ straightforwardly becomes:

$$\delta \lesseqgtr 0 \quad \Longleftrightarrow \quad (v - \hat{v}) \left(\frac{\tau_\mu}{\tau_y} - (\alpha_0 + \alpha_1 v)(\alpha_0 + \alpha_1 \hat{v}) \right) \lesseqgtr 0. \tag{5.8}$$

Hence, forecasters tend to, all else equal, overreact when the perceived and actual weight on private information is sufficiently large (τ_x' and $\hat{\tau}_x$ large, so that v and $\hat{v}$ are large), and when forecasters infer sufficient information from the public signal (τ_y large relative to τ_μ). (Conversely, underreactions occur when the weight on private information is low and the public signal is sufficiently uninformative relative to prior information.)

5.5 Data-consistent Expectations

Unlike the models in Section 4, the forecasts from (5.5) can be consistent with all three stylized facts documented in Section 3. We show this concretely by focusing on our results in Table I ($b > 0$, $\beta < 0$, and $\delta < 0$), where we consider previous period's consensus estimate as the relevant public signal ($\alpha_0 = 0$ and $\alpha_1 = 1$).

Proposition 5. *Suppose $\alpha_0 = 0$ and $\alpha_1 = 1$, such that the public signal y corresponds to previous period's consensus forecast, and consider individual $i \in [0,1]$'s forecast*

$$f_{i2}(\theta) = (1 - k_x)\,\hat{\mathbb{E}}[\theta \mid \mu_i, \hat{y}] + k_x x_i, \quad k_x \in (k_{x,\star}, 1). \tag{5.9}$$

If $\tau_\theta^2 < \tau_x \tau_x'$ and $\hat{\tau}_x = \tau_x$, then there exists a $\bar{\tau}_y > 0$ s.t. for $\tau_y < \bar{\tau}_y$, $b > 0$, $\beta < 0$, and $\delta < 0$.

The first and second result in Proposition 5 ($b > 0$ and $\beta < 0$) resemble those in Proposition 2. On the one hand, because of the dispersion in private information, the average information across forecasters is more precise than any individual forecaster's. This, in turn, causes average forecasts to underreact to the average new information observed ($b > 0$). On the other hand, despite these underreactions at the average level, at the individual level, forecasters instead overrevise their forecasts ($\beta < 0$). This is once more in part due to forecasters' overconfidence in their own private information.

However, where Proposition 5 differs from Proposition 2 is that forecasters' also overreact to the past consensus outcome ($\delta < 0$). These overreactions occur because forecasters' perceived and actual weight on private information are sufficient to ensure that the "perceived under-responsiveness of consensus" dominates its "perceived under-precision".[43] Combined with the dispersion and overconfidence in private information, this then ensures that the forecasts from (5.5) are consistent with all three empirical results documented in Table I.

[43] The reason for the constraint that $\tau_y \leq \bar{\tau}_y$ in Proposition 5 is to limit the magnitude of overresponses to public information, so that estimates of b do not also become negative.

We close this subsection with an additional observation that we also mentioned in the introduction: We note that *underreactions* to endogenous public signals, here exemplified by consensus, naturally arise when the public signal that forecasters observe is sufficiently imprecise. Computing the limit of (5.8) shows then when $\tau_\mu/\tau_y \to \infty$, δ becomes positive.[44] In the next section, we relate this finding to our empirical estimates of δ for other public signals than past consensus outcomes.

Lemma 1. *If $\tau_\mu/\tau_y \to \infty$, then δ in (2.7) eventually becomes strictly positive.*

6 Quantitative Implications

We have shown how our model of overconfidence can be qualitatively consistent with our stylized facts about individual forecasts. Although our model is simple, in this subsection we explore the capacity of the model to also quantitatively match the survey data. We show that our model can account for the baseline estimates in Table I, and that these estimates imply degrees of absolute and relative overconfidence that are in line with auxiliary data. We also test several key implications of our model, and discuss its economic consequences.

6.1 Model Calibration

We use a simulated method of moments procedure to choose parameter values. Normalizing the precision of the fundamental to one, identification of the three parameters τ_x, $\tau_{x'}$ and τ_ξ requires at least three target moments. We choose the individual overrevision and overreaction coefficients β and δ, respectively, documented in Table I. We choose the previous consensus forecast as the benchmark public signal because its structure is simple and known ($\alpha_0 = 0$, $\alpha_1 = 1$ in 5.1), and because its only relationship with future inflation is that of aggregating others' information. We then later show that the estimated model also matches the responses of individual forecast errors to other public signals. Finally, we also include the standard deviation of individual forecast revisions among our target moments.

The criterion we choose to minimize is

$$\Lambda(\tau) \;=\; [\hat{m} - m(\tau)]' W^{-1} [\hat{m} - m(\tau)],$$

where $\hat{m}$ is a vector of target moments from the data and $m(\tau)$ is the vector of simulated moments as a function of the parameter vector $\tau = (\tau_x, \tau_x', \tau_\xi, \tau_\theta)$. As in Proposition 5, we

[44] Notice that (5.8) also shows that $\delta > 0$ when we decrease $\hat{\tau}_x$ sufficiently keeping the degree of relative overconfidence $v/\hat{v}$ constant. This will be important later for Section 6. Furthermore, we note that such underreactions can be so forceful as to maker forecasters underrevise their estimates ($\beta > 0$). This connects to our discussion about this possibility for the EA SPF in Section 3.

Table IV: SMM Estimation: Inflation Forecasts

	β	δ	σ_{fccorr}	β^{mv}	δ^{mv}	$\sqrt{\tau_x}$	$\sqrt{\tau_\xi}$	$\sqrt{\tau_\theta}$	$\sqrt{\tau_x'}$
Data	-0.192	-0.189	0.977	-0.196	-0.191				
Model	-0.184	-0.188	0.968	-0.134	-0.156	0.800	6.667	1.000	1.600

The table presents the values of the target moments (β, δ, and the standard deviation of forecast revisions σ_{fccorr}) for SPF inflation forecasts (first row), and their model estimates (second row). The table also reports the (non-targeted) multivariate coefficients β^{mv} and δ^{mv}, corresponding to column four in Table I, as well as the estimates of the model's precision parameters.

employ the restriction that $\hat{\tau}_x = \tau_x$. Throughout, we use an identity weighting matrix $W = I_4$ for the calibration.[45]

Table IV presents the results for inflation, where for ease of interpretation we report the square root of the precision, the inverse of the standard deviation. Our model is able to capture all three data moments well. We estimate private signals to be rather noisy ($\sqrt{\tau_x} = 0.80$) and the noise in consensus to be small ($\sqrt{\tau_\xi} = 6.67$). At a level of overconfidence that increases the square-root of the perceived precision of private signals by a factor of two, the model predicts well the overrevision of individual forecasts β and the overreaction to past consensus realizations δ. This is consistent with our previous discussion, which illustrated how the combination of a precise consensus and meaningful overconfidence, all else equal, made overreactions more pervasive. Table IV shows that our estimates also capture well the non-targeted multivariate coefficients (β^{mv} and δ^{mv}), while Appendix D documents that the model can also match the data on individual forecasts of output and consumer price inflation.

6.2 Model Evaluation

The parameter estimates in Table IV are chosen to best capture our baseline estimates in Table I. In this subsection, we illustrate how the estimated model also matches several other dimensions of the data that were not included in the calibration. We consider the implied degree of overconfidence, the responses of inflation forecasts to other public signals, and lastly the behavior of the response coefficients β and δ across different data samples.

[45]Previous versions of this paper used a weighting matrix with the inverse of the standard errors on the diagonal and zeros elsewhere. We calculated the standard errors using a bootstrap procedure. However, given the double-clustering of standard errors for β and δ in Table I, the appropriate bootstrap procedure is highly complex. We, therefore, decided for transparency to use a simple identity-weighting matrix instead for Table IV. The estimation results with the alternative weighting matrix are almost identical for the benchmark inflation forecasts, and only slightly changed for the CPI and GDP forecasts.

Table V: Absolute Overconfidence and Density Forecasts

Confidence Bands	*Confidence Level*	
	95 percent	66 percent
SPF Density Implied	0.82***	0.56**
Giordani and Soderlind (2003)	0.72**	0.48**
Model Implied	0.74	0.43

The table shows the implied coverage ratio. The confidence bands from the SPF are derived assuming a normal distribution and are calculated as: mean of inflation individual forecast $\pm$ critical value $\times$ standard deviation. Actual inflation is measured as the percentage change in the GDP Deflator (annual-average). The significance of differences between the nominal confidence level and the actual are assessed using Christoffersen's (1998) test. * $p<0.10$, ** $p<0.05$, *** $p<0.01$. The sample is from 1981Q1 to 2018Q4.

6.2.1 Estimates of Overconfidence

The estimates in Table IV imply a noticeable degree of absolute and relative overconfidence. We now turn to how the implied estimate of absolute overconfidence from Table IV matches well that available from individual density forecasts from the SPF. We then develop a test for our second key assumption of relative overconfidence, and confront it with auxiliary data.

Absolute Overconfidence and the Data: The individual density forecasts of one-year ahead inflation, available in the SPF, allow us to evaluate whether the estimated degree of absolute overconfidence in our model is reasonable. We can do so by comparing the individual forecast densities reported to the SPF with those implied by our estimates.

Table V presents this comparison in the form of *coverage ratios,* describing the percentage of times when inflation outcomes fall inside an individual forecaster's 95 (or 66) percent confidence band. It contrasts the coverage ratios implied by our benchmark estimates in Table IV with those that are estimated from the SPF data (see also Table III). On balance, the implied degree of absolute overconfidence captures well that in the SPF data. Forecasters' 95 percent confidence band has a coverage ratio of only 74 percent, consistent with a sizable amount of absolute overconfidence. This matches closely the SPF estimate of an 82 percent coverage ratio. In fact, Giordani and Söderlind (2003) find very similar degrees of absolute overconfidence to those implied by our model estimates, using a somewhat more advanced estimation method to deduce individual confidence bands from reported forecast densities. We view the estimates in Table V as a key result that corroborates our first main assumption of absolute overconfidence.

Relative Overconfidence and the Data: Density forecasts, such as those in the SPF, can be used to assess the extent of *absolute* overconfidence, or the perceived precision of forecasters' information relative to the truth. In contrast, to assess the extent of *relative* overconfidence, we require information about forecasters' perception of other forecasters' accuracy. This is typically not available, including in the survey data that we have considered so far. In fact, as far as we know, the only exception is the survey of financial executives and forecasters in Germany, carried out by the Centre for European Economic Research (ZEW). It asks its respondents (every month) not only for their own perceptions of (an index) of aggregate activity six months from now, but also for their forecast of the average (or consensus) estimate. Such information can, in turn, be used to construct a test for our second main assumption of relative overconfidence.

Consider the consensus forecast of the fundamental from Section 5 ($\alpha_0 = 0$, $\alpha_1 = 1$ in 5.1)

$$f(\theta) = y = v\theta + \xi, \tag{6.1}$$

and compare it to the consensus estimate that forecasters perceive

$$\hat{f}(\theta) = \hat{y} = \hat{v}\theta + \xi, \tag{6.2}$$

where $v > \hat{v}$ due to forecasters relative overconfidence. Because of the misperception inherent to relative overconfidence, when forecasters are asked to provide a forecast of consensus, they will report a forecast of (6.2) instead of (6.1). As a result, a relationship arises between the average forecast error of consensus, on the one hand, and consensus itself, on the other hand. Specifically, (6.1) and (6.2) imply the linear relationship:

$$f(\theta) - f[f(\theta)] = \alpha + v(\theta - f(\theta)) + (v - \hat{v}) f(\theta) + u, \tag{6.3}$$

where α and u denote a constant and error term respectively. Conditional on the consensus forecast error of the fundamental $\theta - f(\theta)$, a positive relationship arises between the average forecast error of consensus $f(\theta) - f[f(\theta)]$ and consensus $f(\theta)$ itself *if and only if* forecasters exhibit relative overconfidence ($v > \hat{v}$).

Intuitively, when forecasters exhibit relative overconfidence, their forecast errors of consensus $f(\theta) - f[f(\theta)]$ do not only reflect their forecast errors of the underlying fundamental $\theta - f(\theta)$. Instead, because forecasters underestimate the responsiveness of consensus to the fundamental ($v > \hat{v}$), their forecast errors also reflect consensus itself $f(\theta)$.

Table VI provides the estimate of the key coefficient $v - \hat{v}$ in (6.3), partialling out the effect of the forecast error of the fundamental.[46] The estimate in Table VI shows a significant and positive coefficient on consensus, consistent with our second main assumption of relative

[46] The estimate of the coefficient v on the average forecast error of the fundamental in (6.3) is 0.02, and significant at the ten percent level.

Table VI: Relative Overconfidence and ZEW Forecasts

	Conditional Consensus Forecast Error
Constant	-0.0015
	(0.056)
Consensus	0.0367**
	(0.016)
Sample	02M3-19M10
Obs.	193
R^2	0.023

(i) The table shows estimates of $v - \hat{v}$ in (6.3) (Appendix C.2).
(ii) HAC standard errors. * p<0.10, ** p<0.05, *** p<0.01.

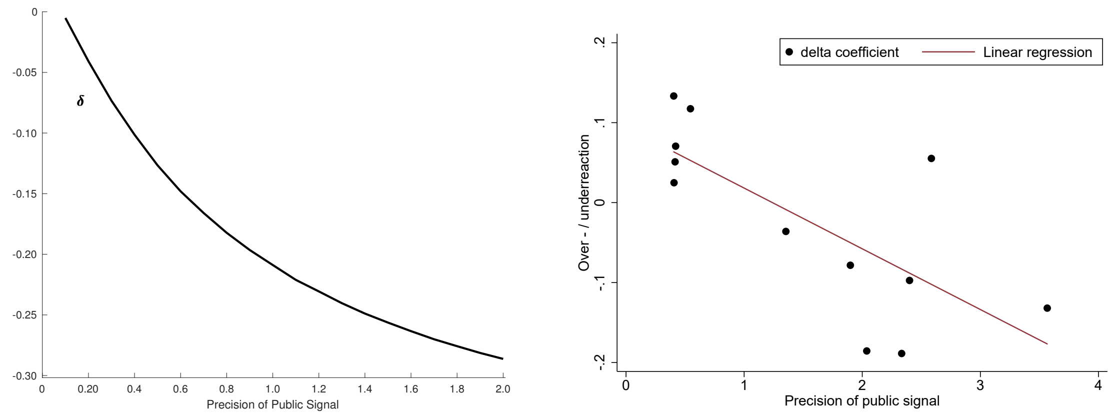
overconfidence. All else equal, forecasters attach more weight to their private information than they perceive others attach. That said, clearly, the estimates in Table VI are not directly comparable to those from our calibrated model. Both the horizon of the forecast and the outcome variable is different. In addition, in reality, forecasters have access to substantially more public information than the sole public signal in (5.3). Consequently, the corresponding estimate of the difference between v and $\hat{v}$ in our calibrated model is somewhat larger than that in Table VI. Notwithstanding this discrepancy, the estimates in Table VI do provide independent validation of the second main assumption of our model, that of relative overconfidence. Finally, we note that the models of generalized overreactions discussed in Section 4, including that of Bordalo *et al.* (2019), are inconsistent with the evidence in Table V and VI.

6.2.2 Over- and Underreactions to Different Public Signals

In this subsection, we discuss the implications of different properties of public signals and of the forecasted variable for individuals' over- and underreactions. Specifically, we analyze how the over- and underreaction coefficient δ in (3.3) changes with respect to the precision of public information and that of the forecasted variable. We in each case compare the comparative statics of our model to estimates from the survey data.

Heterogeneity in Responses to Public Information: A key feature of our empirical results is the heterogeneous responses to public signals, ranging from over- to underreaction ($\delta \lesseqgtr 0$). As the previous section showed, our model is able to replicate this stylized fact. The left-hand panel in Figure 5, in essence, depicts the comparative statics in Lemma 1. It shows that, according to our model, a key parameter governing the responses to public signals is their precision: All else equal, when maintaining other parameters at their benchmark values, our model predicts stronger overreactions to more precise signals. The right-hand panel of Figure 5, by contrast, illustrates the relationship between the precision of individual public signals of one-year ahead inflation and the over- and underreaction coefficient δ, using our estimates in Figure 2. In line with the prediction of our model, we observe stronger overreactions to more precise signals. This leads credence to an important comparative static of our model, and by implication to the notion that overconfidence represents the source of the underlying misperception in forecasters' response to public information.

Figure 4: Overreaction and the Precision of Public Signals



The left panel illustrates variations in δ implied by changes to the precision of consensus τ_ξ relative to its calibrated value from Table I. A value of one on the horizontal axis, therefore, corresponds to a precision of public information equal to that in Table I. By contrast, the right hand panel shows the estimates of δ for different public signals (along the vertical axis), using all of the estimates from Panel (a) and (b) in Figure 2, as a function of the signals' estimated precision (along the horizontal axis). Consistent with Figure 2, we estimate the precision of public signals as the inverse of the variance of an error term. For consensus signals (Panel a in Figure 2), the error term equals the difference between the realized value of one-year ahead inflation and its consensus forecasted value. For other public signals (Panel b in Figure 2), the error terms are instead constructed as the residuals from a linear regression of one-year-ahead inflation onto the public signal in question. To make the precision and δ estimates comparable across series, we focus on the longest common sample available (1981Q1-2016Q4) and standardize the variable over this sample. Notice that this contrasts to Figure 2, in which δ is estimated on the full-sample for each series. Finally, we drop the TIPS and the SCE from the above figure, since these are only available after 2015 (see also the Online Appendix).

Over- and Underreactions and the Great Moderation: Condition (5.8) and Lemma 1 suggest that another key parameter for the magnitude of the response-coefficients δ is the volatility of the series that forecasters try to predict. The more volatile the series is, the weaker the response coefficient is, all else equal. Table VII tests this prediction in the context of the decline in macroeconomic volatility witnessed during the Great Moderation. The Great Moderation saw the standard deviation of inflation fall to less than half its previous value. We also here illustrate the coefficient on individual forecast revisions β. The first line in Table VII

Table VII: Changes in Response Coefficients Pre- vs Post-Great Moderation

	β	δ	$\Delta\beta^{GM}$	$\Delta\delta^{GM}$
Data	-0.192	-0.189	-0.21	+0.09
Model	-0.184	-0.188	-0.14	+0.12

The table shows in its first row the change in the response coefficients β and δ when estimating (3.2) and (3.3) for inflation forecasts using subsamples until (after) 1984, spanning the pre- and post-Great Moderation periods, respectively. The table's second row shows the change in coefficients implied by the model when changing the volatility of inflation and consensus noise in the model by the same amount around their benchmark values, but leaving all other coefficients at their estimated values.

shows that, when estimating the coefficients δ and β in (3.3) and (3.2) on the pre- and post-Great Moderation sample, we find that the magnitude of the former almost doubles, while the latter declines by one third. As the second line of the table shows, when changing the volatility of the fundamental and consensus noise in the model by the same amount around their benchmark values, but leaving all other coefficients at their estimated values, the model captures both changes well. The fact that our model can capture the opposing responses of the coefficient on the overall forecast revisions β and the response to the public consensus signal δ creates confidence that the endogeneity of public signals is important for individual expectations. Simple models of generalized overreactions, such as those considered in Section 4 (e.g. Bordalo *et al.*, 2019), would in contrast predict that both coefficients should respond in the same direction.

6.3 Model Implications

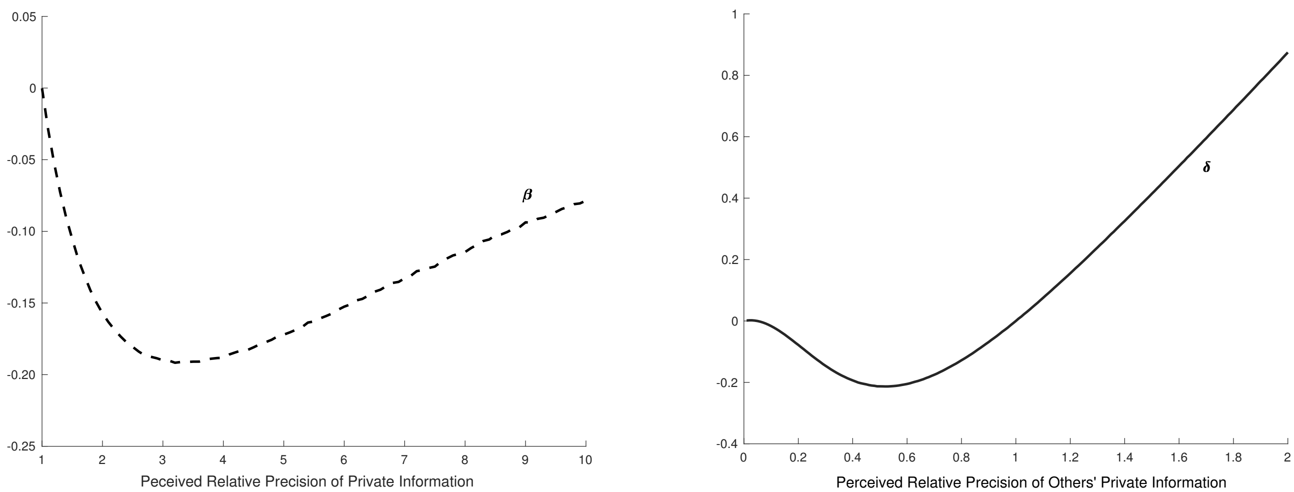
In this section, we discuss how several model implications are affected by changes to the degree of absolute and relative overconfidence. We also here discuss the implications of our model for the distribution of individual forecast errors.

6.3.1 Sensitivity of β and δ to the Degree of Overconfidence

Our implied estimates of absolute and relative overconfidence are clearly specific to our sample of professional forecasters. Other individuals may be more or less overconfident. The left-hand panel of Figure 5 shows how individual forecasters' overall responses to new information (as captured by β) change as we vary the extent of *absolute overconfidence*. We do so by modifying the ratio of the perceived and actual precision of private information (τ_x'/τ_x), but fix all other parameters at their benchmark values. Consistent with Proposition 2, in the absence of overconfidence ($\tau_x'/\tau_x = 1$), neither over- nor underreactions to new information occur on average ($\beta = 0$). As overconfidence then rises, the β-coefficient turns negative, and eventually follows a U-shaped relation with τ_x'/τ_x.

The right-hand panel of Figure 5 instead varies the extent of *relative overconfidence*, as captured by the ratio $(\hat{\tau}_x/\tau_x')$. We see that δ is negative (positive) whenever that ratio is smaller (larger) than one. For the calibrated parameters, our model generically implies overreactions to past consensus outcomes when forecasters perceive others' private information to be less precise than their own.

Figure 5: Sensitivity of Results to Alternative Parameter Choices



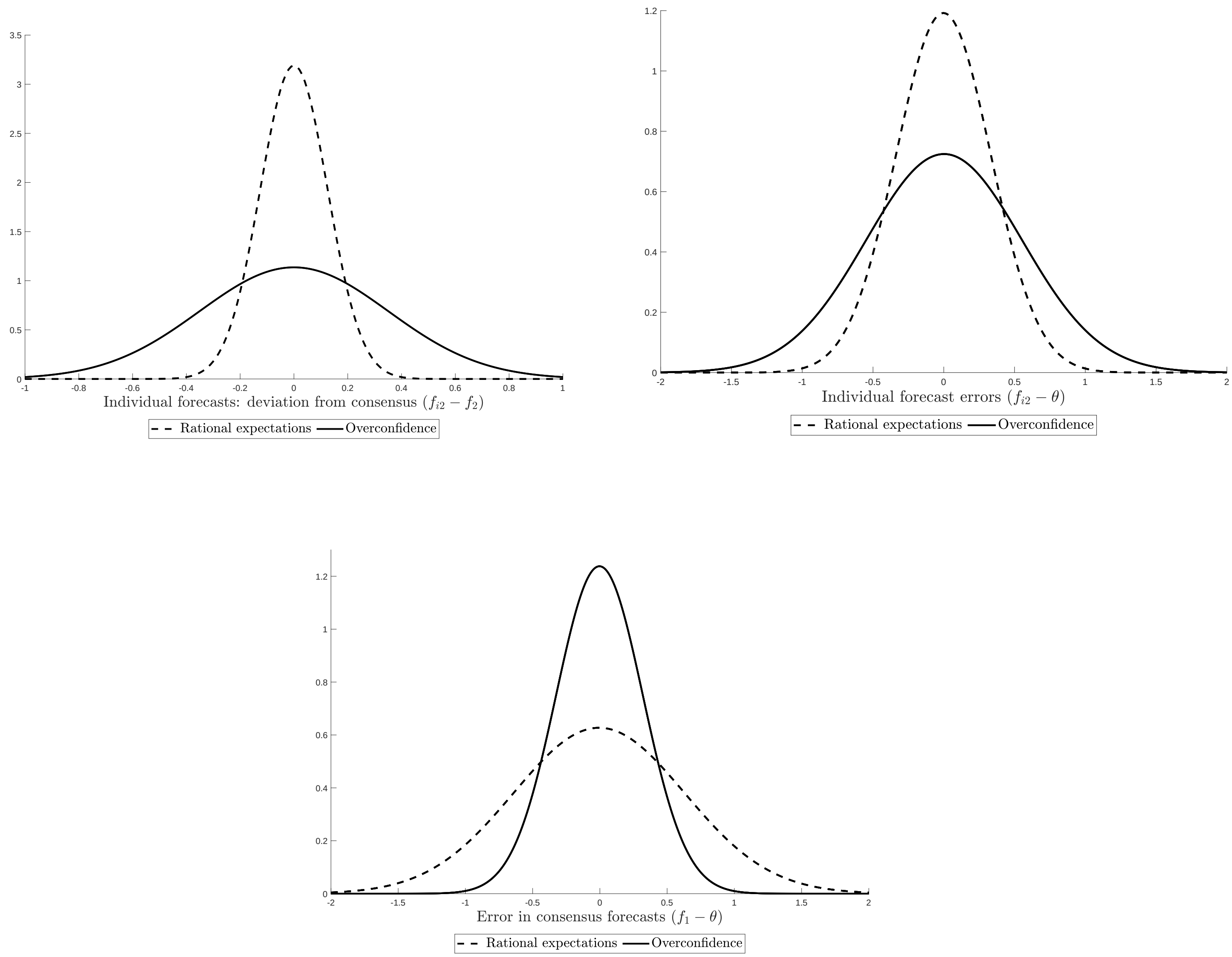
The chart depicts the coefficients δ and β (on the vertical axis) as a function of parameters of the model (along the horizontal axis): The left-hand panel considers the dependence of β on the extent of absolute overconfidence (τ_x'/τ_x). The right-hand panel considers how δ changes with the extent of relative overconfidence $(\hat{\tau}_x/\tau_x')$.

6.3.2 Implications for the Distribution of Forecasts

We conclude this section by exploring auxiliary implications of our model, using the estimates from Table IV. Specifically, we show how overconfident forecasts are substantially more dispersed than their mean-squared optimal counterparts. However, despite this increased dispersion, individual forecast errors from the two cases are remarkably similar. This is because overconfidence also causes endogenous public signals to be more informative. Lastly, we discuss how overconfidence can amplify the effect of public "noise shocks", and how this could have important implications for the role of such shocks in driving business cycle fluctuations

The top left-hand panel in Figure 6 shows the (demeaned) distribution of individual second-period forecasts implied by the model. Compared to rational, mean-squared optimal forecasts, the standard deviation of the overconfident forecast distribution is about three times larger in

Figure 6: The Behavior of Calibrated Individual Forecasts



The top left-hand panel depicts the distribution of the difference between individual second-period forecasts and second-period consensus. It does so for both the overconfidence model and the corresponding mean-squared optimal, rational expectations model. In both cases, we use the parameters listed in Table IV. The top right-hand panel, by contrast, shows the corresponding distribution of individual forecast errors in the two cases. The bottom panel depicts the distribution of the errors in the first-period consensus forecast.

the second period. This is because overconfidence causes individuals to put additional weight on private information. Overconfidence in the precision of private information can thus help explain the a priori puzzling amount of forecast dispersion in macroeconomic forecasts (Muth, 1961, Mankiw *et al.*, 2003, and more recently in Fuster *et al.* 2019).

Importantly, this increase in dispersion does not, however, lead to substantially more imprecise forecasts in equilibrium. The top right-hand panel in Figure 6 shows that the standard deviation of individual second-period forecast errors is only slightly larger in the overconfident case. As a result, forecasters in our model would face difficulty inferring from the precision of their own forecast alone that they were indeed overconfident.[47]

The bottom panel in Figure 6 illustrates that the reason for this close equivalence is that the endogenous public signal (consensus, in this case) is substantially more precise in the overconfident case. Because overconfident forecasters put more weight on private information, the endogenous consensus outcome embeds more of the sum of forecasters' private information, the only truly new information that forecasters can learn from each other. In effect, overconfidence in private information counteracts the standard learning externality that exists in markets with endogenous public information and which causes agents to attach too little weight to private information (e,g. Vives, 1997; Amador and Weill, 2010). That is why, despite the misuse and misinterpretation of information, overconfident forecasters in our model do similarly well to fully rational ones.

A core argument for rational, mean-squared optimal expectations is that such beliefs make agents as well-off as they can be (Brunnermeier and Parker, 2005). However, this rationale for rational expectations relies upon agents being strictly worse off with non-rational beliefs. As Figure 6 shows, this is not necessarily the case in our model. This connects our results with those of Smith (1982), Weibull (1997), and others that attempt to find "group optimal explanations" for individual biases.

Finally, a substantial literature in macroeconomics has explored whether noise shocks to public information can explain business cycle fluctuations (e.g. Chahrour and Jurado, 2018 and the references therein). Because agents in our model can attach more weight to public information than optimal, any such shocks can also have a heightened effect on individual expectations. Compared to a rational model, our model could therefore predict larger responses to public noise shocks. This illustrates one potentially important implication of the combination of absolute and relative overconfidence. Others include: (i) increases in trade in financial assets, due to increases in the dispersion of (relative) beliefs; (ii) "over-shooting" of asset prices in response to public announcements; and (iii) increases in investments into new

[47]Because of mean-squared error preferences, this is equivalent to the statement that they would face difficulty inferring from their own utility alone that they were overconfident.

product lines. We leave these topics, and others, for future research.

7 Concluding Remarks

Expectations are a central determinant of economic allocations. In part because of this central role, a considerable debate has arisen since Muth's (1961) seminal contribution about the best model of expectation formation. Recently, influential evidence has shown that *average* forecasts across a wide variety of surveys are consistent with models of noisy information and rational information use (Coibion and Gorodnichenko, 2015). By contrast, in this paper we have explored the implications of such models for *individual* professional forecasts.

We have demonstrated how the statistical properties of individual inflation forecasts contradict simple versions of noisy rational expectations. Specifically, we have documented two stylized facts: First, individual forecasters' overrevise their macroeconomic expectations. Second, such overrevisions mask evidence of both over- and underreactions to salient public signals. We have shown that such responses violate a basic tenet of noisy rational expectations, the Law of Iterated Expectations, and demonstrated that such violations also contradict several common agency-based and behavioral models of expectation formation.

In place, we have proposed a simple extension of noisy rational expectations, consistent with the stylized facts. We have allowed forecasters to believe that their own private information is not only better than it truthfully is (absolute overconfidence), but also better than that available to others (relative overconfidence). Combined, these biases entail that forecasters both overreact to private information and misperceive the informativeness of endogenous public information that aggregates other agents' private news. We showed that the latter can cause forecasters to both under- and overreact to public information in a manner that is consistent with the data. Lastly, we have demonstrated that our model is not only *qualitatively* consistent with the observed forecast data but also captures key features *quantitatively*, and have validated several of our model's key comparative statics in the data.

We hope that the paper may serve as a stepping stone for further empirical and theoretical research, along similar lines. For example, relative to models of generalized overreactions (e.g. Bordalo *et al.*, 2019), our model predicts both over- and underreactions to *endogenous* public information. However, one important difference is that our model does not predict overreactions (or underreactions for that matter) to purely *exogenous* public signals. While difficult to test with macroeconomic data, where most relevant public signals reflect the outcomes of people's actions and expectations, there might be other, including experimental settings, that could use such contrasting implications to compare the different theories.[48]

[48] A second important difference is that models of generalized overresponses are not necessarily recursive, unlike those in Proposition 5 that inherit the recursivity of the rational model. Bordalo *et al.* (2019), for

Finally, our model has illustrated how simple behavioral biases can combine with the endogeneity of public information to create rich patterns of predictability in individual forecast errors. This basic idea is more general than our particular forecaster application. In future research, it would, for example, be valuable to consider asset price and business cycle implications of richer descriptions of absolute and relative overconfidence in markets where traders learn from prices. This would also have the advantage of creating further testable predictions.

example, assume that previous information enters expectations in the form of a rational prior every period. This makes the timing of news important: current expectations, and thus forecasts, overreact to current news, but the effect vanishes in the next forecast when priors are reset to their rational values. This implies negative serial correlation in forecast revisions, and stronger overresponses to more recent information.

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Appendix A: Alternative Explanations

Appendix A.1 Reputational Considerations

We follow Ehrbeck and Waldmann (1996), but extend their setup to allow for public information. There is a continuum of measure one of forecasters $i \in [0,1]$ with prior beliefs $\theta \sim \mathcal{N}\left(\mu_i, \tau_\theta^{-1}\right)$. Each forecaster i observes a private signal

$$x_i^j = \theta + \epsilon^j, \quad \epsilon^j \sim \mathcal{N}\left[0, \left(\tau_x^j\right)^{-1}\right],$$

where $j = \{1,2\}$ and $\tau_x^1 > \tau_x^2$. In line with Ehrbeck and Waldmann (1996), we assume that $i \in [0, 1/2[$ observe x_i^1, while $i \in [1/2, 1]$ observe x_i^2. In addition, each forecaster observes y in (2.2).

We consider linear equilibria, in which forecaster i's forecasting rule is characterized by

$$f_i = (1-w)\,\mathbb{E}[\theta \mid \mu_i, y] + w x_i^j,$$

where (potentially) $w \neq w_\star$. Following the same steps as in Ehrbeck and Waldmann (1996) shows that if we only consider Nash equilibria in which able forecasters are frank (and forecasters care only about the posterior odds of being viewed as able by their clients), then $w > w_\star$ for $i \in [1/2, 1]$. It now follows from Proposition 2 that across all forecasters $i \in [0,1]$ $b > 0$, $\beta < 0$, but $\delta = 0$.

Appendix A.2 Generalized Overreactions

The forecasting rule in (4.7) can be re-stated as:

$$f_i = \mu_i + (1+\chi)\, k_\star \left(w_x x_i + w_y y - \mu_i\right) \tag{1}$$

where $w_x = \frac{\tau_x}{\tau_\theta + \tau_x + \tau_y}$ and $w_x + w_y = 1$. We now show that for all $\chi > 0$ we have that $\delta < 0$.

To do so, consider

$$\begin{aligned} \delta \times y &= \mathbb{E}\left[\theta - f_i \mid y\right], \\ &= \mathbb{E}\left[\theta - f_i^{RE} + f_i^{RE} - f_i \mid y\right] = \mathbb{E}\left[f_i^{RE} - f_i \mid y\right] \end{aligned}$$

where f_i^{RE} denotes the noisy rational expectation forecast (i.e. 1 with $\chi = 0$). Thus,

$$\delta \times y = -\chi k_\star \mathbb{E}\left[w_x x_i + w_y y \mid y\right] = -\chi k_\star \left(w_x \frac{\tau_y}{\tau_\theta + \tau_y} + w_y\right) y,$$

and we conclude that $\delta = -\chi k_\star \left(w_x \frac{\tau_y}{\tau_\theta + \tau_y} + w_y\right) < 0$.

Appendix A.3 Strategic Complementarity Extension

The orthogonality of individual forecast errors to public information follows from a similar argument to that which establishes Proposition 2. Since $f = \int_0^1 f_i di$, we can re-write (4.8) as

$$f_i = \mathbb{E}\left\{ r \sum_{i=0}^{\infty} (1-r)^i \bar{\mathbb{E}}^i\left[\theta\right] \mid \mu_i, x_i, y \right\}, \tag{2}$$

where $\bar{\mathbb{E}}[\theta] = \int_0^1 \mathbb{E}[\theta \mid \mu_i, x_i, y]\, di$ and $\bar{\mathbb{E}}^i[\theta] = \int_0^1 \mathbb{E}\left\{\bar{\mathbb{E}}^{i-1}[\theta] \mid \mu_i, x_i, y\right\} di$. But now notice that from the Law of Iterated Expectations:

$$\mathbb{E}[f_i \mid y] = r\sum_{i=0}^{\infty}(1-r)^i\mathbb{E}[\theta \mid y] = r\frac{1}{1-(1-r)}\mathbb{E}[\theta \mid y] = \mathbb{E}[\theta \mid y].$$

Hence,

$$\delta \times y = \mathbb{E}[\theta - f_i \mid y] = 0,$$

which completes the statement in the main text.

Appendix A.4 Trembling-hand Noise

Let $\tilde{f}_i \equiv f_i + e_i$ denote forecaster i's stated, trembling-hand forecast, where $e_i \sim \mathcal{N}\left(0, \tau_e^{-1}\right)$. His actual forecast is still equal to f_i. We then have that

$$\tilde{\beta} \equiv \mathbb{C}ov\left(\theta - \tilde{f}_i, \tilde{f}_i - \mu_i\right)\mathbb{V}\left[\tilde{f}_i - \mu_i\right]^{-1} = \frac{\mathbb{C}ov\left(\theta - f_i, f_i - \mu_i\right) - \tau_e^{-1}}{\mathbb{V}[f_i - \mu_i] + \tau_e^{-1}}.$$

Thus,

$$\tilde{\beta} = \beta\frac{\tau_e}{\tau_e + \mathbb{V}[f_i - \mu_i]^{-1}} - \frac{\mathbb{V}[f_i - \mu_i]^{-1}}{\tau_e + \mathbb{V}[f_i - \mu_i]^{-1}}.$$

However,

$$\tilde{\delta} \equiv \mathbb{C}ov\left(\theta - \tilde{f}_i, y\right)\mathbb{V}[y]^{-1} = \mathbb{C}ov\left(\theta - f_i - e_i, y\right)\mathbb{V}[y]^{-1} = \delta.$$

Appendix A.5 Empirics & Alternative Reputational Considerations

Table 1: Revisions and Errors in Inflation

	(1) Absolute Forecast Error
Absolute Forecast Revision	0.264***
	(0.0377)
Constant	0.850***
	(0.0195)
R^2	0.039
N	5016

(i) Double-clustered standard errors in parentheses
(ii) * $p < .1$, ** $p < .05$, *** $p < .01$

Appendix B: A Model of Overconfidence

Proof of Proposition 3: We have from (5.6) that

$$\begin{aligned}
\delta \times y = \mathbb{E}\left[\theta - f_{i2} \mid y\right] &= (1-k_x)\left(\mathbb{E}\left[\theta \mid y\right] - \mathbb{E}\left[\hat{\mathbb{E}}\left[\theta \mid \mu_i, \hat{y}\right] \mid y\right]\right) \\
&= (1-k_x)\,\mathbb{E}\left\{\mathbb{E}\left[\theta \mid \mu_i, y\right] - \hat{\mathbb{E}}\left[\theta \mid \mu_i, \hat{y}\right] \mid y\right\} \neq 0. \qquad (3)
\end{aligned}$$

It follows that the conditional expectation in (3) based upon the realized public signal y is

$$\mathbb{E}\left[\theta \mid \mu_i, y\right] = \kappa y + (1-\kappa)\mu_i, \quad \kappa = \frac{\eta^2 \tau_y}{\tau_\mu + \eta^2 \tau_y} \times \frac{\hat{\eta}}{\eta} \qquad (4)$$

while the overconfident expectation based upon $\hat{y}$ is

$$\hat{\mathbb{E}}\left[\theta \mid \mu_i, \hat{y}\right] = \hat{\kappa} y + (1-\hat{\kappa})\mu_i, \quad \hat{\kappa} = \frac{\hat{\eta}^2 \tau_y}{\tau_\mu + \hat{\eta}^2 \tau_y} \times 1. \qquad (5)$$

Thus, δ is proportional to the simple difference between $\hat{\kappa}$ and κ

$$\delta = \Delta(\kappa - \hat{\kappa}), \qquad (6)$$

where $\Delta \equiv (1-k_x)\left(1 - \frac{\tau_\mu}{\tau_\theta + \tau_\mu}\right) \in (0,1)$.[1] $\square$

Proof of Proposition 4: The proof proceeds in two steps. We first show that $\beta < 0$ and $b > 0$ when $\tau_y \to_+ 0$. We thereafter show that if $\tau_\theta^2 > \tau_x \tau_x'$ then $\delta \to_- 0$ when $\tau_y \to_+ 0$. Continuity of all three coefficients in τ_y establishes the rest of the proof.

Step 1: Individual forecast errors and revisions are, when $\tau_y \to 0$, respectively:

- $\theta - f_{i2} = (1-2w_x)(\theta - \mu) - w_x(\epsilon_{i2} + \epsilon_{i1})$, $\quad w_x = \frac{\tau_x'}{\tau_\mu + \tau_x'}$
- $f_{i2} - \mu_i = (2w_x - v_x)\theta + (w_x - v_x)\epsilon_{i1} + w_x \epsilon_{i2} + (1 - 2w_x - v_x)\mu$.

It follows that

$$\beta \propto \mathbb{C}ov\left(\theta - f_{i2}, f_{i2} - \mu_i\right) = \frac{\tau_\theta \tau_x' \left(\tau_x - \tau_x'\right)}{\tau_x\left(\tau_\theta + \tau_x'\right)\left(\tau_\theta + 2\tau_x'\right)^2} < 0.$$

Similarly, since $f_2 = \int_0^1 f_{i2} di$ and $\bar{\mu}_i = \int_0^1 \mu_i di$,

- $\theta - f_2 = (1-2w_x)(\theta - \mu)$
- $f_2 - \bar{\mu}_i = (2w_x - v_x)\theta + (1 - 2w_x - v_x)\mu$.

Thus,

$$b \propto \mathbb{C}ov\left(\theta - f_2, f_2 - \bar{\mu}_i\right) = \frac{\tau_\theta \tau_x'}{\left(\tau_\theta + \tau_x'\right)\left(\tau_\theta + 2\tau_x'\right)^2} > 0.$$

Step 2: Section 5 in the main text showed that

$$\mathbb{E}\left[\theta - f_{i2} \mid y\right] = (1-w)\left(\mathbb{E}\left[\theta \mid y\right] - \mathbb{E}\left[\hat{\mathbb{E}}\left[\theta \mid \mu_i, y_s\right] \mid y\right]\right).$$

It follows that:[2]

[1] We note that μ_i is equivalent to the observation of the perceived private signal $\theta + \varepsilon_i$, $\varepsilon_i \sim \mathcal{N}\left(0, \tau_\mu^{-1}\right)$. Thus, $\mathbb{E}\left\{\mathbb{E}\left[\theta \mid \mu_i, y\right] \mid y\right\} = \left[\kappa + (1-\kappa)\Delta\right] y$ and $\mathbb{E}\left\{\hat{\mathbb{E}}\left[\theta \mid \mu_i, \hat{y}\right] \mid y\right\} = \left[\hat{\kappa} + (1-\hat{\kappa})\Delta\right] y$, where $\varphi \equiv \frac{\tau_\mu}{\tau_\theta + \tau_\mu}$. Combined, this provides us with $\delta = (1-k_x)(1-\varphi)(\kappa - \hat{\kappa})$ in (6).

[2] We here disregard the irrelevant constant μ.

- $\mathbb{E}[\theta \mid y] = \alpha_0 y, \quad \alpha_0 = \left(\frac{v_\star}{v}\right) \times \frac{v^2\tau_y}{\tau_\theta + v^2\tau_y}$
- $\hat{\mathbb{E}}[\theta \mid \mu_i, y_s] = \alpha_1\mu_i + (1-\alpha_1)\, y, \quad \alpha_1 = \frac{\tau_\mu}{\tau_\mu + v_\star^2\tau_y}, \quad \tau_\mu = \tau_\theta + \tau_x'$
- $\mathbb{E}\left[\hat{\mathbb{E}}[\theta \mid \mu_i, y_s] \mid y\right] = \alpha_1 v_x \alpha_0 y + (1-\alpha_1)\, y.$

Thus,

$$\delta \propto \alpha_0 + \alpha_1 - 1 - \alpha_1\alpha_0 v_x \equiv \alpha.$$

It is clear that $\delta \to 0$ when $\tau_y \to_+ 0$. Yet, a few simple but tedious derivations also show that

$$\frac{\partial \alpha}{\partial \tau_y}_{|\tau_y \to 0} = \frac{\tau_x'\left(\tau_x' - \tau_x\right)\left(\tau_x\tau_x' - \tau_\theta^2\right)}{\left(\tau_\theta + \tau_x\right)^2\left(\tau_\theta + \tau_x'\right)^2},$$

so that $\frac{\partial\delta}{\partial\tau_y}\,|_{\tau_y\to 0} < 0$ if $\tau_\theta^2 > \tau_x\tau_x'$. Thus, if $\tau_\theta^2 > \tau_x\tau_x'$ there exists a threshold $\bar{\tau}_y \in \mathbb{R}_+$ such that $\delta < 0$.

Necessary and Sufficient Conditions for $\delta < 0$*:* Direct calculations show that

$$\delta \times y = (1-w)\left(\mathbb{E}[\theta \mid y] - \mathbb{E}\left[\hat{\mathbb{E}}[\theta \mid \mu_i, \hat{y}] \mid y\right]\right) \tag{7}$$

$$= \frac{(1-w)(1-\frac{v}{\hat{v}})\hat{v}^2\tau_\xi}{\left(\frac{v}{\hat{v}}\right)^2\hat{v}^2\tau_\xi + \tau_\mu}\left[\frac{v}{\hat{v}} + \left(1 + \frac{v}{\hat{v}}\right)\frac{\tau_\mu}{\tau_\mu + \hat{v}^2\tau_\xi}\,\frac{\frac{v}{\hat{v}}(1-\frac{v}{\hat{v}})\hat{v}^2\tau_\xi - \tau_\theta}{\left(\frac{v}{\hat{v}}\right)^2\hat{v}^2\tau_\xi + \tau_\theta}\right] y. \tag{8}$$

Thus, $\delta < 0$ whenever $\tau_\theta < \frac{\tau_x'}{\tau_x}v_\star^2\tau_\xi$, since $v/\hat{v} > 1$ and the term inside the bracket in (8) is positive while that in front of is negative. □

Proof of Lemma 1: Follows immediately from taking limits of (5.8). □

Appendix C: Quantitative Implications

Appendix C.1 Additional Calibrations

Table 2: SMM Estimation: GDP Growth Forecasts

	β	δ	β^{mv}	δ^{mv}	σ_{fccorr}	$\sqrt{\tau_x}$	$\sqrt{\tau_\xi}$	$\sqrt{\tau_\theta}$	$\sqrt{\tau_x'}$
Data	-0.193	0.209	-0.223	0.228	1.248				
Model	-0.283	0.062	-0.292	0.089	1.235	0.333	2.000	1.000	1.111

The table presents the values of the target moments β, δ and the standard deviation of forecast revisions σ_{fccorr} in SPF data for GDP growth forecasts (first row), and their model estimates (second row). The table also reports the (non-targeted) multivariate coefficients β^{mv} and δ^{mv}, corresponding to column (4) in Table I, and the estimates of the model's precision parameters (reported here in terms of the square root of the precision, the inverse of the standard deviation), $\sqrt{\tau_x}$, $\sqrt{\tau_\xi}$, $\sqrt{\tau_\theta}$ (normalized to one), and $\sqrt{\tau_x'}$.

Table 3: SMM Estimation: CPI Inflation Forecasts

	β	δ	β^{mv}	δ^{mv}	σ_{fccorr}	$\sqrt{\tau_x}$	$\sqrt{\tau_\xi}$	$\sqrt{\tau_\theta}$	$\sqrt{\tau_x'}$
Data	-0.294	-0.439	-0.330	-0.494	0.777				
Model	-0.297	-0.226	-0.172	-0.187	0.822	1.429	100.000	1.000	9.524

The table presents the values of the target moments β, δ and the standard deviation of forecast revisions σ_{fccorr} in SPF data for CPI inflation forecasts (first row), and their model estimates (second row). The table also reports the (non-targeted) multivariate coefficients β^{mv} and δ^{mv}, corresponding to column (4) in Table I, and the estimates of the model's precision parameters (reported here in terms of the square root of the precision, the inverse of the standard deviation), $\sqrt{\tau_x}$, $\sqrt{\tau_\xi}$, $\sqrt{\tau_\theta}$ (normalized to one), and $\sqrt{\tau_x'}$.

Online Appendix to

"Forecaster (Mis-)Behavior"

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This appendix complements the analysis in our paper "Forecaster (Mis-) Behavior". The Appendix contains two sections and is organized as follows: Appendix A contains the description of the data used to document the empirical results in Section 3 and 6 of our paper. Appendix B, in turn, describes the corresponding estimation results in detail.

Appendix A: Data Description

This appendix describes the construction of the variables used in regressions (3.1) to (3.3).

Appendix A.1: Forecast Errors and Revisions

We construct forecast errors and forecast revisions as in Coibion and Gorodnichenko (2015). Here, we focus on our main variable of interest, inflation, but the construction of the variables is identical for other series. For the individual regressions (3.2) and (3.3), we construct individual forecast errors for quarter $t+h$ ($\pi_{t+h} - f_{it}\pi_{t+h}$) as the difference between the first release of the inflation outcome in $t+h$ and the h-quarter ahead individual forecast (and equivalently for the average regression 3.1 using the consensus forecast $f_t\pi_{t+h}$). The forecast revision (average or individual) is, in turn, the difference between the (consensus or individual) period t forecast of inflation in $t+h$ and the $t+h$ forecast published in period $t-1$.

Appendix A.2: Forecaster Surveys

We estimate our main regressions (3.1), (3.2), and (3.3) using data from three different surveys.

From the US SPF, we use forecasts of GDP deflator inflation (constructed from quarterly forecasts of the level of the GDP deflator, series identifier PGDP), real output growth (constructed from quarterly forecasts of the level of real output, series identifier RGDP[1]), and CPI inflation (concatenating quarter growth rate forecasts, series identifier CPI).[2] Note that the level

[1] Prior to 1981, real output forecasts were constructed from nominal GDP forecasts and PGDP

[2] For example, for the GDP deflator, the formula for the one-year ahead inflation forecast is $100 * (PGDP5_t - PGDP1_t)/PGDP1_t$. The formula for the forecast revision in turn is: $rev = 100 * (PGDP5_t - PGDP1_t)/PGDP1_t - 100 * (PGDP6_{t-1} - PGDP2_{t-1})/PGDP2_{t-1}$.

of real output and its deflator is unknown in period t.

From the Euro Area Survey of Professional Forecasters, we consider forecasts of real output growth and HICP inflation, in addition to their forecast revisions, and construct them in a similar fashion as those from the US SPF. We also use individual forecasts for CPI inflation from the biannual Livingstone Survey (constructed from the level forecasts for the current period, as well as 6 and 12-months ahead forecasts). Here, the forecast horizon we consider is 6 months, and revisions are constructed as the difference between the period t forecast for inflation 6-months ahead and the period $t-1$ forecast for inflation between 6 and 12-months ahead.

Lastly, to construct Table V, we also consider the individual density forecasts of GDP deflator inflation from the US SPF (series identifier PRPGDP). These document survey respondents' perceived probability that the percentage change in the annual average of the US GDP deflator in a given calendar year falls within a certain range. We use only the one-year ahead forecasts that are constructed in Q4 of each year, to make our analysis as consistent as possible with our previous estimates. To estimate the individual confidence intervals, we employ a normal approximation to the stated individual probability distributions, and estimate these as described in the main text.

Appendix A.3: Data on Outcome Variables

As explained in the body of our paper, and standard in the literature, we compare forecasts to outcomes as they are first released (Croushore, 1993). Data for first-release realizations of inflation and output are taken from the real-time databases maintained by the St. Louis Federal Reserve Bank (ALFRED) and the European Central Bank (ECB's Statistical Data Warehouse).

Appendix A.4: Data on Public Signals

The public signals y used in regression (3.3) are the most recent realization of variable y that is available at the time forecasters make their period t forecast.

For example, the deadline for SPF responses (late in the second to third week of the middle month of each quarter) is set such that respondents know the Bureau of Economic Analysis' advance report of the National Income and Product Accounts from the previous quarter at the time of their response. Forecasters also know the (first release) consumer price inflation outcome for the first month of the survey quarter, and, importantly, the consensus forecast of the previous round of the survey. We obtain the release dates for all public signals from BLOOMBERG. Financial market variables are at daily frequency. We use the observation on the last day in the month preceding the survey, e.g. January 30 for the February survey.

Apart from $t-1$ consensus forecasts from the surveys described above, we also consider average expectations from a number of other surveys. Specifically, we consider median expectations for 12-months ahead inflation from the University of Michigan's Survey of Consumers (obtained from the FRED database),[3] the Federal Reserve Bank of New York's Survey of Con-

[3] See: http://www.sca.isr.umich.edu/

sumer Expectations (obtained from the New York Fed website),[4] the European Commission's Survey of Consumer Expectations (obtained from EUROSTAT),[5] Consensus Economics forecasts (obtained with permission from Consensus Economics),[6] and lastly the Blue Chip Survey of Economic indicators (obtained from Wolters Kluwer).[7]

As non-survey public signals, we consider 12-month percentage changes in the nominal effective exchange rate for US and the Euro Area, respectively, taken from the Bank for International Settlements' homepage (series identifier "Narrow Nominal Effective Exchange Rate"); and import price indices, taken from the FRED database and the ECB's Statistical Data Warehouse for the US and the Euro Area, respectively (series identifier MXP for the US and STS.M.I8.N.IMPX for the Euro Area). We also consider three signals embodied in financial prices: the percent year-over-year change in the S&P 500 stock market index (US) and the DAX (Euro Area); the implied inflation from 10-year US Treasury inflation-protected securities (TIPS) for the US, and from inflation-protected swaps at one year maturity for the Euro Area; and the term spread between the implied rate on 10-year and 3-months treasury securities (US and Germany for the Euro Area). All financial data are obtained from BLOOMBERG. Finally, we also consider the main measure of the US unemployment rate (from the Bureau of Labor Statistics, calculated as total unemployment as a fraction of the total labor force, obtained from the FRED database, series identifier UNRATE), and the Euro Area unemployment rate (from EUROSTAT, series identifier UNE_RT_M).

Appendix A.5: Other Data

Finally, to test for relative overconfidence (Table VI), we employ data from the Centre for European Economic Research (ZEW)'s Financial Market Report. Specifically, we use the monthly forecast of the consensus estimate of the six-month ahead ZEW index of economic activity, taken from BLOOMBERG (series identifiers GRZEWI and GRZECURR).

[4]See: https://www.newyorkfed.org/microeconomics/sce

[5]See: https://ec.europa.eu/info/business-economy-euro/indicators-statistics/economic-databases/business-and-consumer-surveys_en

[6]See: https://www.consensuseconomics.com/

[7]https://lrus.wolterskluwer.com/store/product/blue-chip-economic-indicators/

Appendix B: Empirical Results

This Appendix presents additional estimates of our three regression equations:

$$\pi_{t+h} - f_{t|t+h} = a + b\left(f_{t|t+h} - f_{t-1|t+h}\right) + v_t \tag{A1}$$

$$\pi_{t+h} - f_{it|t+h} = \alpha_i + \beta\left(f_{it|t+h} - f_{it-1|t+h}\right) + v_{it} \tag{A2}$$

$$\pi_{t+h} - f_{it|t+h} = \alpha_i + \delta y_t + v_{it} \tag{A3}$$

using alternative sample periods, forecast horizons, industry-groupings, forecast variables, and surveys (Section B.1), as well as a variety of public signals other than consensus (Section B.2).

Appendix B.1: Alternative Surveys and Samples

Table 1: Survey of Professional Forecasters: CPI inflation

	(1) Average	(2) Individual	(3) Individual	(4) Individual
Forecast revision	0.270			
	(0.250)			
Forecast revision		-0.294***		-0.330***
		(0.0965)		(0.0905)
Previous consensus			-0.439***	-0.494***
			(0.0703)	(0.0795)
Constant	-0.217**	-0.271***	1.037***	1.138***
	(0.104)	(0.0939)	(0.224)	(0.237)
R^2	0.014	0.205	0.232	0.261
N	138	3594	4695	3594

Note: Column one presents estimates of b; two and three of β and δ, respectively; column four of both simultaneously, using SPF forecasts for CPI inflation. Robust (column one) or robust double-clustered (remaining columns) standard errors in parentheses. ∗ p<.1,∗∗ p<.05,∗∗∗ p<.01

Table 2: Survey of Professional Forecasters: GDP growth

	(1) Average	(2) Individual	(3) Individual	(4) Individual
Forecast revision	0.612**			
	(0.245)			
Forecast revision		-0.193***		-0.223***
		(0.0601)		(0.0561)
Previous consensus			0.209	0.228
			(0.163)	(0.161)
Constant	-0.131	-0.294**	-0.933*	-0.984*
	(0.112)	(0.114)	(0.552)	(0.536)
R^2	0.053	0.177	0.139	0.189
N	184	5119	6882	5119

Note: Column one presents estimates of b; two and three of β and δ, respectively; column four of both simultaneously, using SPF forecasts for GNP growth until 1991 and GDP thereafter. GNP was calculated from nominal forecasts until 1982. Robust (column one) or robust double-clustered (remaining columns) standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01

Table 3: SPF: Alternative Sample Period (Philadelphia Federal Reserve Sample)

	(1) PGDP	(2) PGDP	(3) PGDP	(4) CPI	(5) CPI	(6) CPI	(7) RGDP	(8) RGDP	(9) RGDP
Average forecast revision	0.581			0.210			0.494		
	(0.228)			(0.439)			(0.346)		
Forecast revision		-0.372***			-0.274			-0.107	
		(0.0515)			(0.174)			(0.137)	
Previous consensus			-0.375***			-0.518***			-0.504**
			(0.101)			(0.157)			(0.237)
Constant	-0.287***	-0.333***	0.509**	-0.120	-0.159	1.161***	-0.0769	-0.166*	1.237*
	(0.0480)	(0.0448)	(0.223)	(0.121)	(0.108)	(0.396)	(0.0953)	(0.0971)	(0.705)
R^2	0.255	0.323	0.286	0.132	0.167	0.164	0.210	0.198	0.225
N	3782	3028	3782	3819	3039	3819	3929	3140	3929

Note: Column one, four, and seven, present estimates of b; column two, five, and eight of β; column three, six, and nine for δ, for forecasts of GDP deflator inflation (PGDP), consumer price inflation (CPI) and real GDP growth (GDP). The sample period starts in 1990 Q4, when the Federal Reserve Bank of Philadelphia took over the management of the SPF. Robust (column one, four, and seven) or robust double-clustered (remaining columns) standard errors in parentheses. * $p<.1$,** $p<.05$,*** $p<.01$

Table 4: Survey of Professional Forecasters: Semi-annual Forecast Horizon ($h = 2$)

	(1) Aggregate	(2) Individual	(3) Individual	(4) Individual
Forecast revision	0.617***			
	(0.146)			
Individual forecast revision		-0.287***		-0.285***
		(0.0513)		(0.0508)
Previous consensus			-0.0581	0.0958
			(0.0803)	(0.0773)
Constant	-0.0322	0.0189	-0.247	-0.296
	(0.0551)	(0.0636)	(0.215)	(0.246)
R^2	0.171	0.245	0.159	0.250
N	199	5679	7420	5679

Note: Column one presents estimates of b; column two and three of β and δ for 6m-ahead semi-annual GDP inflator inflation ($h = 2$). Robust (column one) or robust double-clustered (remaining columns) standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01

Table 6: Livingstone Survey of Forecasters: CPI Inflation

	(1) Average	(2) Individual	(3) Individual	(4) Individual
Average forecast revision	-1.155			
	(0.754)			
Forecast revision		-0.518***		-0.529***
		(0.0890)		(0.0857)
Previous consensus			-0.316**	-0.439
			(0.114)	(0.274)
Constant	-0.351	-0.211	0.773	0.935
	(0.284)	(0.180)	(0.852)	(0.788)
R^2	0.090	0.268	0.110	0.279
N	49	1291	1687	1291

Note: Column one presents estimates of b; two and three of β and δ, respectively; column four of both simultaneously, for 12m ahead GDP growth forecasts from the Livingstone survey. Robust (column one) or robust double-clustered (remaining columns) standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01

Table 5: Survey of Professional Forecasters: Inflation Forecasts from Different Industries

	(1) Financial, average	(2) Financial	(3) Financial	(4) Non-financial, average	(5) Non-financial	(6) Non-financial
Forecast revision	0.621**	-0.367***		0.364	-0.366***	
	(0.266)	(0.0603)		(0.223)	(0.0452)	
Previous consensus			-0.316***			-0.300**
			(0.0921)			(0.116)
Constant	-0.202***	-0.272***	0.430**	-0.363***	-0.363***	0.320
	(0.0581)	(0.0379)	(0.199)	(0.0590)	(0.0477)	(0.265)
R^2	0.042	0.445	0.429	0.025	0.233	0.172
N	108	1235	1603	103	1684	2061

Note: Columns one and three present estimates of β; columns two and four δ, using 12-month ahead SPF GDP deflator forecasts by forecasters in the financial industry (columns one and two), and in non-financial industries (columns four to six). Robust (column 1, 4) or robust double-clustered (remaining columns) standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01

Table 7: Livingstone Survey of Forecasters: GDP growth

	(1) Average	(2) Individual	(3) Individual	(4) Individual
Average forecast revision	0.826*			
	(0.438)			
Forecast revision		-0.113		-0.132
		(0.147)		(0.187)
Previous consensus			-1.215***	-1.263***
			(0.393)	(0.427)
Constant	0.114	0.0443	3.491***	3.563***
	(0.204)	(0.173)	(1.154)	(1.242)
R^2	0.110	0.191	0.282	0.300
N	49	1428	1820	1428

Note: Column one presents estimates of b; two and three of β and δ, respectively; column four of both simultaneously, for 12m ahead GDP growth forecasts from the Livingstone survey. Robust (column one) or robust double-clustered (remaining columns) standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01

Table 8: Livingstone Survey of Forecasters: CPI inflation, Sub-groups

	(1) Academic	(2) Academic	(3) Comm. Banking	(4) Comm. Banking	(5) Consulting	(6) Consulting	(7) Inv. Banking	(8) Inv. Banking	(9) Nonf. Business	(10) Nonf. Business
Forecast revision	-0.792***		-0.236		-0.416**		-0.442***		-0.598***	
	(0.127)		(0.184)		(0.154)		(0.115)		(0.0755)	
Prev. consensus		-0.317		-0.417		-0.135		-0.642***		-0.362
		(0.321)		(0.342)		(0.372)		(0.214)		(0.369)
Constant	-0.0305	0.955	-0.190	1.057	-0.342*	0.112	-0.305**	1.777**	-0.128	0.908
	(0.176)	(0.900)	(0.169)	(0.965)	(0.195)	(0.987)	(0.125)	(0.655)	(0.177)	(1.015)
R^2	0.296	0.080	0.216	0.171	0.290	0.139	0.430	0.273	0.223	0.055
N	202	234	197	254	275	387	215	298	301	398

Note: Columns 1, 3, 5, 7, and 9 presents estimates of β, while columns 2, 4, 6, 8, and 10 of δ for 12m ahead forecasts for CPI inflation from the Livingstone Survey. Robust double-clustered standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01

Table 9: Euro Area SPF: GDP Forecasts

	(1) GDP	(2) GDP	(3) GDP	(4) GDP
Average forecast revision	0.638***			
	(0.206)			
Forecast revision		0.367**		0.318*
		(0.168)		(0.162)
Previous consensus			-0.797***	-0.701***
			(0.178)	(0.166)
Constant	0.0934	-0.0435	1.354***	1.323***
	(0.131)	(0.132)	(0.326)	(0.318)
R^2	0.173	0.087	0.092	0.138
N	2401	2401	2401	2401

Note: Column one presents estimates of b; two and three of β and δ, respectively; column four of both simultaneously, for 12m ahead forecasts for GDP growth from the Euro Area SPF. Robust (column one) or robust double-clustered (remaining columns) standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01

Table 10: Euro Area SPF: HICP Forecasts

	(1) HICP	(2) HICP	(3) HICP	(4) HICP
Average forecast revision	0.782*			
	(0.400)			
Forecast revision		-0.0672		-0.0770
		(0.154)		(0.160)
Previous consensus			-0.535	-0.546
			(0.687)	(0.702)
Constant	0.209*	0.132	1.081	1.094
	(0.106)	(0.0969)	(1.201)	(1.222)
R^2	0.146	0.100	0.106	0.108
N	2388	2388	2388	2388

Note: Column one presents estimates of b; two and three of β and δ, respectively; column four of both simultaneously, for 12m ahead forecasts for HICP inflation from the Euro Area SPF. Robust (column one) or robust double-clustered (remaining columns) standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01

Appendix B.2: Alternative Measures of Consensus and Other Public Signals

Table 11: SPF: Alternative Measures of Consensus Forecasts for Inflation

	(1) PGDP	(2) PGDP	(3) PGDP	(4) PGDP	(5) PGDP	(6) PGDP
Previous consensus	-0.372**					
	(0.159)					
Michigan survey		0.0381				
		(0.112)				
SCE			-0.308***			
			(0.0466)			
Consensus Economics				-0.132**		
				(0.0630)		
Blue Chip					0.495**	
					(0.189)	
Livingstone						0.0818***
						(0.0233)
Constant	-0.00286	-0.281***	-0.290***	-0.264***	0.000550	-0.263***
	(0.0791)	(0.0446)	(0.0596)	(0.0542)	(0.0986)	(0.000882)
R^2	0.219	0.257	0.494	0.255	0.262	0.243
N	7189	5529	778	2375	3764	1973

Note: Estimates of δ, using 12-month ahead forecasts for inflation from the SPF, and different measures of consensus forecasts. Robust double-clustered standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01.

Table 12: Survey of Professional Forecasters: Alternative Public Signals for Inflation

	(1) PGDP	(2) PGDP	(3) PGDP	(4) PGDP	(5) PGDP	(6) PGDP	(7) PGDP	(8) PGDP	(9) PGDP
Tips	0.119*								
	(0.0584)								
Lag		-0.443***							
		(0.162)							
Exchange rate			0.323***						
			(0.0799)						
Import prices				0.0553					
				(0.0496)					
Oil price					0.0510				
					(0.0422)				
Unemployment rate						0.460***			
						(0.0904)			
Financial Markets							-0.0974		
							(0.0762)		
Stock price								-0.531***	
								(0.0969)	
Term spread									0.173***
									(0.0533)
Constant	-0.113	-0.0276	0.0710	-0.284***	-0.361***	0.0748	-0.346***	0.0840	-0.238***
	(0.0757)	(0.0760)	(0.0774)	(0.0425)	(0.0393)	(0.0742)	(0.0396)	(0.0757)	(0.0429)
R^2	0.294	0.230	0.242	0.240	0.268	0.278	0.240	0.280	0.255
N	575	7169	7460	4211	5274	7460	5021	7460	5880

Note: Estimates of δ, using 12-month ahead forecasts for inflation from the SPF, and different public signals than consensus forecasts. Robust double-clustered standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01.

Table 13: SPF: CPI Forecasts and Alternative Measures of Consensus Forecasts

	(1) CPI	(2) CPI	(3) CPI	(4) CPI	(5) CPI	(6) CPI
Previous consensus	-0.560***					
	(0.0896)					
Michigan survey		-0.653***				
		(0.161)				
SCE			-0.457			
			(0.313)			
Livingstone				-0.0230		
				(0.122)		
CSE					-0.208**	
					(0.101)	
Blue Chip						0.742***
						(0.126)
Constant	-0.300***	-0.341***	-0.633**	-0.143	-0.166	-0.397***
	(0.0883)	(0.0936)	(0.240)	(0.157)	(0.143)	(0.124)
R^2	0.232	0.263	0.316	0.137	0.147	0.257
N	4695	4695	404	1798	2026	2359

Note: Estimates of δ, using 12-month ahead forecasts for CPI inflation from the SPF, and different measures of consensus forecasts. Robust double-clustered standard errors in parentheses. ∗ p<.1,∗∗ p<.05,∗∗∗ p<.01.

Table 14: SPF: CPI Forecasts and Alternative Public Signals

	(1) CPI	(2) CPI	(3) CPI	(4) CPI	(5) CPI	(6) CPI	(7) CPI	(8) CPI	(9) CPI
Tips	0.347								
	(0.179)								
Lag		-0.150							
		(0.116)							
Exchange rate			0.223**						
			(0.0924)						
Import prices				-0.145					
				(0.140)					
Oil price					-0.183				
					(0.131)				
Unemployment rate						0.000127			
						(0.112)			
Financial Markets							-0.392***		
							(0.118)		
Stock price								-0.173*	
								(0.0903)	
Term spread									0.130
									(0.0979)
Constant	-0.475**	-0.185**	-0.268***	-0.141	-0.261***	-0.264***	-0.252***	-0.266***	-0.253***
	(0.150)	(0.0919)	(0.0905)	(0.102)	(0.0880)	(0.0919)	(0.0914)	(0.0913)	(0.0886)
R^2	0.460	0.146	0.208	0.142	0.203	0.184	0.188	0.195	0.192
N	197	4517	4695	3873	4695	4695	4620	4695	4695

Note: Estimates of δ, using 12-month ahead forecasts for CPI inflation from the SPF, and different public signals than consensus forecasts. Robust double-clustered standard errors in parentheses. * p<.1,** p<.05,*** p<.01.

Table 15: Livingstone Survey: CPI Forecasts and Alternative Measures of Consensus Forecasts

	(1) CPI	(2) CPI	(3) CPI	(4) CPI	(5) CPI	(6) CPI
Previous consensus	-0.633***					
	(0.173)					
SPF		-1.307***				
		(0.175)				
Michigan survey			-0.747***			
			(0.101)			
SCE				-0.763***		
				(0.0627)		
Consensus Economics					-0.375***	
					(0.0654)	
Blue Chip						0.633***
						(0.173)
Constant	-0.445***	-1.152***	-0.385***	-0.650***	-0.143***	-0.445***
	(0.104)	(0.146)	(0.0435)	(0.0367)	(0.0138)	(0.104)
R^2	0.110	0.129	0.121	0.512	0.119	0.110
N	1687	1687	1687	126	1687	1687

Note: Estimates of δ, using 12-month ahead forecasts for CPI inflation from the Livingstone survey, and different measures of consensus. Robust double-clustered standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01.

Table 16: Livingstone Survey: Alternative Public Signals for Inflation

	(1) CPI	(2) CPI	(3) CPI	(4) CPI	(5) CPI	(6) CPI	(7) CPI	(8) CPI	(9) CPI
Tips	0.659***								
	(0.107)								
Lag		0.0778							
		(0.0502)							
Exchange rate			0.394***						
			(0.0372)						
Import prices				0.100***					
				(0.0285)					
Oil price					0.0288				
					(0.0287)				
Unemployment rate						-0.0206			
						(0.0388)			
Financial Markets							-0.278***		
							(0.0952)		
Stock price								-0.230***	
								(0.0572)	
Term spread									0.164***
									(0.0410)
Constant	-0.856***	-0.0341*	-0.0128***	-0.0621***	-0.0644***	-0.0622***	-0.227***	-0.128***	-0.0109
	(0.0309)	(0.0193)	(0.00484)	(0.000556)	(0.000265)	(0.00357)	(0.0558)	(0.0159)	(0.0133)
R^2	0.683	0.105	0.143	0.107	0.104	0.103	0.108	0.110	0.110
N	73	1687	1687	1687	1687	1687	1687	1687	1687

Note: Estimates of δ, using 12-month ahead forecasts for CPI inflation from the Livingstone Survey, and different public signals. Robust double-clustered standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01.

Table 17: Euro Area SPF: HICP Forecasts and Alternative Consensus Forecasts

	(1) CPI	(2) CPI	(3) CPI
Previous consensus	-0.0920		
	(0.127)		
Consumer exp.		-0.275	
		(0.167)	
Fin. Market exp.			-0.200
			(0.147)
Constant	-0.0146	-0.0140	-0.0308
	(0.134)	(0.126)	(0.139)
R^2	0.059	0.123	0.085
N	2764	2764	2642

Standard errors in parentheses

* $p < .1$, ** $p < .05$, *** $p < .01$

Note: Estimates of δ, using 12-month ahead forecasts for HICP inflation from the Euro Area SPF, and different measures of consensus. Robust double-clustered standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01.

Table 18: Euro Area SPF: HICP Forecasts and Alternative Public Signals

	(1) CPI	(2) CPI	(3) CPI	(4) CPI	(5) CPI	(6) CPI	(7) est7
Lag	-0.0747						
	(0.114)						
Exchange rate		0.133					
		(0.165)					
Import prices			0.0214				
			(0.144)				
Oil Prices				0.0773			
				(0.136)			
Unemp rate					0.223		
					(0.184)		
Equity price						-0.262**	
						(0.121)	
Term spread							-0.390***
							(0.125)
Constant	-0.0146	-0.0149	-0.101	-0.0145	-0.0132	-0.0168	-0.0149
	(0.134)	(0.133)	(0.165)	(0.135)	(0.134)	(0.129)	(0.123)
R^2	0.057	0.068	0.049	0.057	0.093	0.118	0.200
N	2764	2764	2206	2764	2764	2764	2764

Standard errors in parentheses

* $p < .1$, ** $p < .05$, *** $p < .01$

Note: Estimates of δ, using 12-month ahead forecasts for HICP inflation from the Euro Area SPF, and different public signals. Robust double-clustered standard errors in parentheses. ∗ p<.1,∗∗ p<.05,∗∗∗ p<.01.

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